Finding Elliptical Shapes in an Image Using a Pyramid Architecture

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Abstract

The problem of finding elliptical shapes in an image on a pyramid architecture using moment properties is considered. Based on the moment principle, the proposed method can be employed to determine the five parameters of an elliptical shape in a given image, including the coordinates of the center of the ellipse, the length of minor and major axes, and the rotation angle of the major axis. A simulation program is also described.

Key Words: Distributed computing, ellipse, image processing, moments, parallel algorithms, pattern recognition, pyramid architecture.

1 Introduction

Several methods have been used to detect elliptical objects. One approach is based on the edge detection [14] and another approach is based on the use of Hough transform [3]. In this paper moment of an image has been used as a feature for ellipse detection. Hu [6] has derived results showing the algebraic invariant of two dimensional moments. Alt [2] has applied Hu's results for the recognition of letters and numerals. Dudani et al. have applied moment invariants for aircraft identification [5]. Teague [12] extended Hu's idea by introducing the *orthogonal moment set* to recover images from moments. This idea is based on the theory of *orthogonal polynomials*. Teague introduced *Zernike moments* which allow for easy construction of independent moment invariants for an arbitrarily high order. Recently there have been some recent efforts in the area of orthogonal moments. Reddi [11] proposed a simpler construction method using radial and angular moments. Abu-Mostafa and Psaltis [1] introduced the notion of *complex moments* and used them to derive moment invariants. Moments are very important in image detection, and contain information about an object which can be used in finding the location and orientation of that object [4, 13, 16]. From the point of view of pattern recognition, moment invariants are considered reliable features if their values are insensitive to the presence of image noise.

In this paper we have adopted Hu's technique because of its limited memory resources utilized, which is of paramount importance for base level processors in the pyramid architecture. For detecting an elliptical shape in an image a pyramid topology has been proposed. This architecture is suitable for high-speed image processing where its simple geometry adapts naturally to many types of problems [8, 9, 10]. Section 2, summarizes properties of moments. In Section 3, moments for ellipse and their properties are described. Section 4 describes an algorithm for ellipse detection Section 5 presents simulation program, and Section 6 gives the conclusion.

2 Moments

To compute moments of an image, a large number of multiplications and additions are required. In this paper, we present a new algorithm that shows the advantages of using a pyramid architecture for reducing the time of calculations and the required storage.

Given a two dimensional density distribution function $f(x, y)$, *(p+q) order* moments are defined as:

$$m_{pq} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x^p y^p f(x, y) dxdy \ where \ p, q = 0, 1, 2, ...$$

The integration extends over the domain of f. For an image with limited area in the $x - y$ plane, the integration extends over the area of the image. The second order moments about x and y axes, i.e., $p = 0$, $q = 2$ and $p = 2$, $q = 0$ are called *moment of inertia.* For $p = 1$ and $q = 1$ the moment is called *product of moment of inertia.*

Definition 1: The center of gravity, *centroid*, of a pattern has coordinates $\overline{x}$ and $\overline{y}$, that are given by: $\overline{x} = m_{10}/m_{00}$ and $\overline{y} = m_{01}/m_{00}$

Central moments are:

$$\mu_{pq} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x - \overline{x})^p (y - \overline{y})^q f(x, y) dx dy$$

A uniqueness theorem states that if $f(x, y)$ is piecewise continuous and has nonzero values only in a finite section of the $x - y$ plane, then moments of all orders exist and the moment sequence m_{pq} uniquely determines $f(x, y)$. This theorem states the detection power of moments in pattern recognition. To tackle the problem of translation, scale change, and rotation caused by the image formation process, a set of seven moment invariants which are invariant under these three transformations have been derived [6]. These invariants are functions of second and third central moments, and they are expressed in terms of m_{pq} moments. Thus, the efficient calculation of m_{pq} is important.

For a binary image where each cell can be 0 or 1, the double integral for moment calculation is approximated by a summation. Therefore, the $(p + q)$ order moment is approximated as:

$$m_{pq} = \sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij}^p y_{ij}^q$$

x_{ij} and y_{ij} are the coordinates of a pixel and mn is the total number of pixels.

Central moments are approximated as:

$$\mu_{pq} = \sum_{i=1}^{n} \sum_{j=1}^{m} (x_{ij} - \overline{x})^p (y_{ij} - \overline{y})^q$$

3 Moments Calculation for Ellipses

In this section, we consider some of the properties of moments in an elliptical object. As shown in Figure 1, we assume $2a$ and $2b$ are the length of major and minor axes of an ellipse, and S the area of an ellipse is given by πab. Several important properties of an ellipse are described below:

Lemma 1: In an ellipse the second moment is minimum along the major axis and maximum along the minor axis; these moments are given by $b^3 a\pi/4$ and $a^3 b\pi/4$.

Proof: In parametric form an ellipse is represented as: $x = a\cos t$ and $y = b\sin t$ where $0 \leq t \leq 2\pi$. Also, we have $dx = -a\sin t dt$ and $dy = b\cos t dt$.

The moment m_{02} for an ellipse can be calculated as follows (using Green's Theorem):

$$m_{02} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} y^2 dx dy = \oint_{ellipse} y^2 x dy = \int_0^{2\pi} ab^3 \cos^2 t \sin^2 t dt = ab^3\pi/4 = Sb^2/4$$

By interchanging a with b, m_{20} is given by $a^3b\pi/4 = Sa^2/4$ (S is the area of an ellipse). Since we have $b < a$, then $m_{20} < m_{02}$.

We make the assumption that the origin of coordinate system is located at the centroid of an ellipse (see Figure 1). From this figure, x' and y', the new values of x and y in the rotated coordinate system are calculated as follows:

$x' = x\cos\theta - y\sin\theta$ and $y' = x\sin\theta + y\cos\theta$

Let $m_{pq\theta}$ denote the $(p+q)$ order moment in the direction of angle θ with respect to x axis. If we substitute x' and y' in the equations for m_{02} and m_{20} in Section 2, we derive the following relations:

$m_{20\theta} = m_{20}\cos^2\theta + m_{02}\sin^2\theta - 2m_{11}\sin\theta\cos\theta$

$m_{02\theta} = m_{20}\sin^2\theta + m_{02}\cos^2\theta + 2m_{11}\sin\theta\cos\theta$

From Lemma 1, we infer that second moments are minimum along one axis and maximum along the other axis. In order to calculate the value of θ, the derivative of $m_{20\theta}$ with respect to θ will be calculated and set to zero. This is given by:

$2\sin\theta\cos\theta m_{02} - 2\sin\theta\cos\theta m_{20} - 2\cos 2\theta m_{11} = 0$

After taking the derivative, we have:

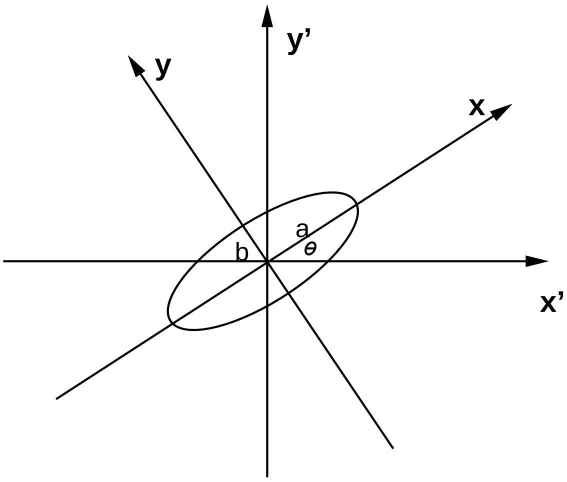

Figure 1: An ellipse in a rotated coordinate system

$\tan 2\theta = \frac{2m_{11}}{m_{02}-m_{20}} \Rightarrow \theta = \frac{1}{2}\tan^{-1}(\frac{2m_{11}}{m_{02}-m_{20}})$

The lengths of major and minor axes in an ellipse are derived by the relations $m_{20} = \pi ba^3/4$ and $m_{02} = \pi ab^3/4$. Thus we have:

$a = \sqrt[4]{\frac{4\sqrt{m_{20}^3}}{\pi\sqrt{m_{02}}}}$ and $b = \sqrt[4]{\frac{4\sqrt{m_{02}^3}}{\pi\sqrt{m_{20}}}}$

4 Moments Calculation on a Pyramid Architecture

In this section parallel calculation of moments on a pyramid architecture are considered.

A pyramid computer with the base size of n is an SIMD machine that is constructed from $(1/2)\log n + 1$ levels of a two-dimensional mesh connected processor array, where the Lth level, $0 \leq L \leq (1/2)\log n$, is a two-dimensional mesh connected

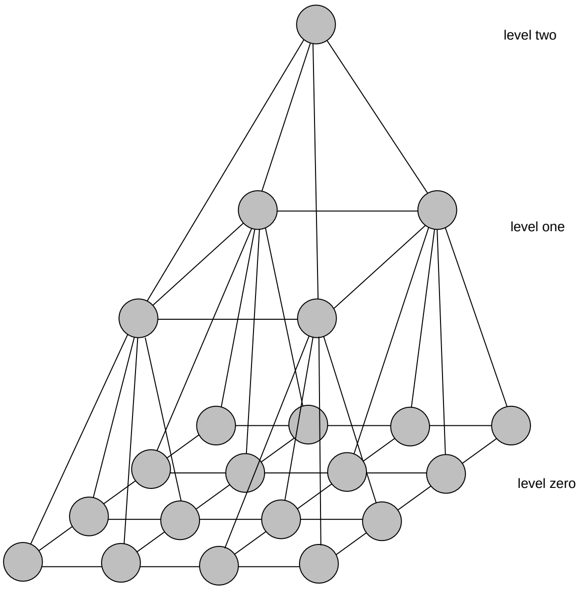

Figure 2: A 4 by 4 base pyramid

processor array of size $\frac{n}{4^L}$. A mesh connected computer of size n is a collection of n processing elements arranged in a $\sqrt{n} * \sqrt{n}$ grid, where each processing element except for those along the border, is connected to its four neighbors. Figure 2 illustrates a pyramid with a 4-by-4 base configuration.

4.1 Moments Algorithm

In the following we assume number of pixels in an image is equal to the number of processors at the base of the pyramid (i.e., n). Each processor at level zero (base) stores x_{ij} and y_{ij}, the x and y coordinates of a pixel p_{ij} in its local memory and the value of pixel p_{ij} can be 0 or 1. Moments Algorithm (MA) uses both mesh and tree connections of pyramid and has four phases. During the first three phases moments

m_{10}, m_{01}, m_{20}, m_{02}, and m_{11} will be calculated.

$$m_{pq} = \sum_{i=1}^{\sqrt{n}} \sum_{j=1}^{\sqrt{n}} x_{ij}^{p} y_{ij}^{q}$$

$m_{00} = n$,

Where $p, q \in \{0, 1, 2\}$.

During Phase one processors at the base will transfer their coordinates to the parent processors at level one and calculate sum for all x_{ij} coordinates and y_{ij} coordinates sequentially. This process is repeated for higher levels. At the end, the apex processor has the final values for the sum of x_{ij} and y_{ij}. In Phase two processors at the base calculate x_{ij}^2 and y_{ij}^2 and send results to the next level for addition and this process is repeated for higher levels. In Phase three each processor at the base calculates the product of $x_{ij} y_{ij}$ and sends the result to its parent processor for summation. This process is repeated at the higher levels.

In Phase four apex processor will find the center of the image by calculating $\overline{x}$ and $\overline{y}$. By using formulas derived in Section 3, direction of the axes, θ, and the length of the major and minor axes (i.e., $2a$ and $2b$) are calculated next.

Figure 3: The 16x16 image created by bitmap editor.

4.2 Algorithm Complexity

In this section, we describe the number of operations needed in different phases of the algorithm. we assume the transfer time and arithmetic operation for $b = \lceil \log \sqrt{n} \rceil$ bits of data are equal. Since all calculations are done in a pipeline fashion, the total time complexity in all four phases is proportional to the number of levels in the pyramid, i.e., $O(\log n)$. The exact number of operations-communication and arithmetic-for all phases is $6 \log n + 17$ steps or $\Theta(\log n)$ complexity for the computation time.

5 Simulation Program

The simulation program for detecting ellipse was written in C language. Figure 3 illustrates a 16 by 16 image created by *bitmap editor* under X windows [15]. Larger version of this image was used to calculate the moments for the rest of entries in Table 1.

The bitmap file of the image in Figure 3 is illustrated in the Figure 4 and is called portable bitmap(pbm). In this pbm file 1 means *black* and 0 means *white*. The simulation program consists of two major parts: *Simulation Controller* and

```
0000000000000000
0000010000000000
0000100000000000
0000100000000000
0001000000000000
0001000000000000
0010000111000000
0000001000100000
0000010000010000
0000010000010000
0000010000010000
0000001000100000
0000000111000000
0000000000000000
0011111111111100
0000000000000000
```

Figure 4: The pbm file of a 16x16 image.

Algorithm Controller. The Simulation Controller is used for scheduling. It offers a menu to the user to select the size of a pyramid. The program has a global clock called *Simulation Clock* and increased whenever the processors at the same level terminate execution. Therefore, at the end of the program, Simulation Clock denotes the number of times that processors activated during execution of the program.

Table 1 illustrates the computer simulation results for different size binary images using Moments algorithm.

Figure 5 represents the relationship between image size and simulation clock for Moments algorithm.

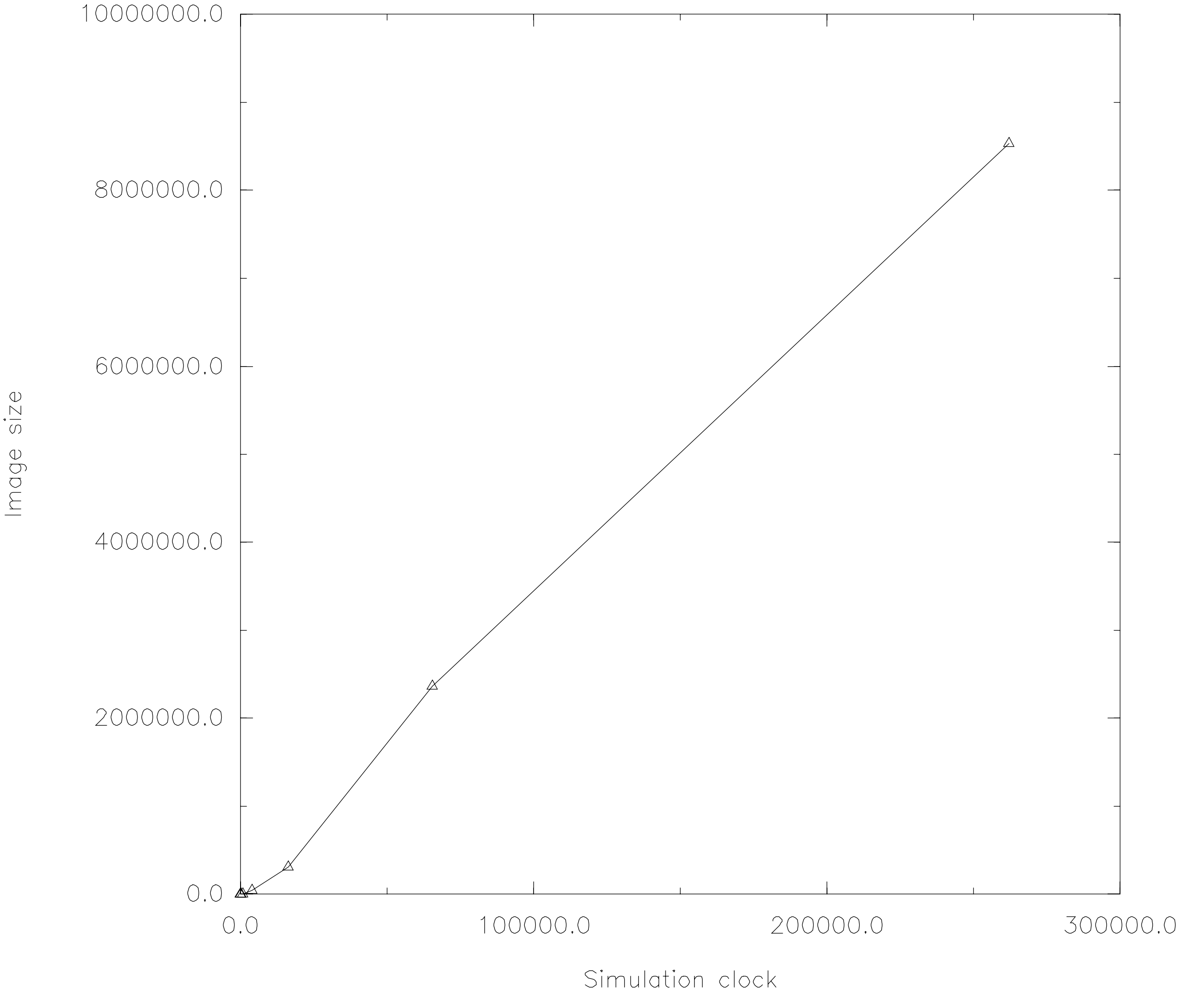

Figure 5: Simulation clock for Moments algorithm

Image size	*Moments algorithm*
8 *by* 8	76
16 *by* 16	680
32 *by* 32	5,432
64 *by* 64	42,176
128 *by* 128	312,534
256 *by* 256	2,367,190
512 *by* 512	8,533,678

Table 1: Simulation clock for ellipse detection algorithm

6 Conclusion

In this paper, we described an algorithm for calculating five parameters of an ellipse using moment properties on the pyramid architecture. Our result from simulation program proves the usefulness of a pyramid architecture in image processing and pattern recognition. An interesting extension of this algorithm is to explore the effect of noise in calculating the five parameters.

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