

## EMBEDDING MESHES INTO THE STAR-CONNECTED CYCLES INTERCONNECTION NETWORK

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### ABSTRACT

The star-connected cycles (SCC) graph was recently proposed as an attractive interconnection network for parallel processing, using a star graph to connect cycles of nodes. This paper presents embeddings of  $p$ -dimensional meshes into an  $n$ -dimensional SCC graph ( $1 \leq p \leq n$ ), which have load 1, expansion 1, and dilation ranging from 1 to  $3 + 2\lfloor(n-1)/2\rfloor + 2\lfloor n/4\rfloor$  (the dilation varies among mesh dimensions). We show that, despite the fact that the dilation of our embeddings increases with  $n$ , a small communication slowdown can be obtained if high-bandwidth links are used within the cycles of the SCC graph.

### KEYWORDS

Interconnection networks, mesh embedding, parallel processing, star-connected cycles graph.

### INTRODUCTION

The star graph is considered an interesting interconnection network for parallel processing, featuring smaller degree and diameter than a hypercube of comparable size (Akers et al., 1987). However, star graphs pose some difficulties for implementing large scalable parallel computers, which is due to their variable-degree topology. A bounded-degree variant of the star graph, which is referred to as the *star-connected cycles (SCC) graph*, was recently introduced to overcome such difficulties (Latifi et al., 1993). SCC graphs require only three I/O ports per node, are suitable for area-efficient VLSI layouts, can efficiently execute algorithms such as broadcasting, and capture many of the attractive characteristics of star graphs (Latifi et al., 1995).

This paper presents a technique by which SCC graphs can run a broad class of algorithms originally devised for mesh-connected parallel computers (e.g., numerical analysis, image processing, sorting, and matrix algorithms). We do so by simulating a  $p$ -dimensional mesh into an  $n$ -dimensional SCC graph via an *embedding*. Through the remainder of this section, we give general definitions of embeddings and their related parameters.

### Definitions

Let  $G_p$  be a  $p$ -dimensional graph with hierarchical structure, such that  $G_{p+1}$  is obtained recursively from  $c_p$  copies of  $G_p$ . Several graphs belonging to the class of *Cayley graphs* have this recursive

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decomposition property, such as the hypercube and the star graph (Akers and Krishnamurthy, 1989). The links connecting the  $c_p$  copies of  $G_p$  that exist within  $G_{p+1}$  are referred to as  $(p+1)^{th}$ -dimension links. We denote the set of nodes and the set of links of  $G_p$  by  $V(G_p)$  and  $E(G_p)$ , respectively. An *embedding* of  $G_p$  into  $H_n$ , which we denote by  $F : G_p \mapsto H_n$ , is a mapping of  $V(G_p)$  into  $V(H_n)$  and of  $E(G_p)$  into paths of  $H_n$ .  $G_p$  and  $H_n$  are respectively referred to as the *guest* and the *host* of  $F$  (Leighton, 1992). The *node image* of  $F$  is  $F(G_p) = \{F(u) : u \in V(G_p)\}$ . The *load* of  $F$  is the maximum number of nodes of  $G_p$  that are mapped to any single node of  $H_n$ , and is denoted by  $\lambda(F)$ . The *expansion* of  $F$  is  $X(F) = |V(H_n)|/|V(G_p)|$ . The *dilation* of  $F$  is  $d(F) = \max\{dist_H(F(u), F(v)) : (u, v) \in E(G_p)\}$ , where  $dist_H(a, b)$  is the distance in  $H_n$  between two vertices  $a$  and  $b$  of  $H_n$ . The *dilation* of  $F$  along the  $j^{th}$  dimension of  $G_p$  is  $d_j(F) = \max\{dist_H(F(u), F(v)) : (u, v) \in E_j(G_p)\}$ , where  $E_j(G_p)$  denotes the set of  $j^{th}$ -dimension links of  $G_p$ ,  $1 \leq j \leq p$ . Hence,  $d(F) = \max\{d_j(F) : 1 \leq j \leq p\}$ .  $F$  is a *variable-dilation embedding* if for at least two different dimensions  $j, k$  of  $G_p$ ,  $d_j(F) \neq d_k(F)$ . The *dilation vector* of  $F$  is  $\underline{d(F)} = [d_1(F), d_2(F), \dots, d_p(F)]$ . The *average dilation* of  $F$  is:

$$d_{avr}(F) = \frac{1}{p} \sum_{j=1}^p d_j(F) \quad (1)$$

## BACKGROUND

### The Star Graph ( $S_n$ )

An  $n$ -dimensional star graph ( $S_n$ ) contains  $n!$  nodes which are labeled with the  $n!$  possible permutations of  $n$  distinct symbols. A node  $\pi = p_1 p_2 \dots p_i \dots p_n$  is connected to  $(n-1)$  distinct nodes, which are respectively labeled with permutations  $\pi_i = p_i p_2 \dots p_{i-1} p_1 p_{i+1} \dots p_n$ ,  $2 \leq i \leq n$  (i.e.  $\pi_i$ 's label is obtained by exchanging the first and the  $i^{th}$  symbol of  $\pi$ 's label) (Akers et al., 1987; Akers and Krishnamurthy, 1989). The link connecting  $\pi$  and  $\pi_i$  is an  $i^{th}$ -dimension link of  $S_n$ .

### The Star-Connected Cycles Graph ( $SCC_n$ )

An  $n$ -dimensional SCC graph ( $SCC_n$ ) is obtained by replacing each node of  $S_n$  with a ring of  $(n-1)$  nodes, namely a *supernode*. The connections between nodes inside the same supernode are referred to as *local links*. Each supernode is connected to  $(n-1)$  adjacent supernodes, using *lateral links* according to the topology of  $S_n$ . Figure 1 shows  $SCC_4$ . Although not shown in Fig. 1, we assume that an  $i^{th}$ -dimension lateral link is labeled  $i$ . The nodes in each ring are identified by a label  $\langle i, \pi \rangle$ , where  $2 \leq i \leq n$  and  $\pi$  is a permutation of  $n$  symbols (in this paper, we use as symbols the integers  $\{1, 2, \dots, n\}$ ). Then, two nodes  $\langle i, \pi \rangle$  and  $\langle i', \pi' \rangle$  are connected by a link ( $\langle i, \pi \rangle, \langle i', \pi' \rangle$ ) in  $SCC_n$  iff either: 1) ( $\langle i, \pi \rangle, \langle i', \pi' \rangle$ ) is a local link, i.e.  $\pi = \pi'$  and  $\min(|i - i'|, n - 1 - |i - i'|) = 1$ , or 2) ( $\langle i, \pi \rangle, \langle i', \pi' \rangle$ ) is a lateral link, i.e.  $i = i'$  and  $\pi$  differs from  $\pi'$  only in the first and  $i^{th}$  symbols, such that  $\pi(1) = \pi'(i)$  and  $\pi(i) = \pi'(1)$ .  $SCC_n$  is a regular, vertex-symmetric graph containing  $|V(SCC_n)| = (n-1) \cdot n!$  nodes. The degree of  $SCC_n$  is  $\delta(SCC_n) = n-1$  (for  $n \leq 3$ ), and  $\delta(SCC_n) = 3$  (for  $n \geq 4$ ). The diameter of  $SCC_n$  is (Latifi et al., 1993):

$$\phi(SCC_n) = \begin{cases} 6, & \text{if } n = 3 \\ 2 \left( \left\lfloor \frac{n-1}{2} \right\rfloor \right)^2 + \left\lfloor \frac{3(n-1)}{2} \right\rfloor + 2 \left\lfloor \frac{n}{2} \right\rfloor - 2, & \text{if } n \neq 3 \end{cases} \quad (2)$$

### Basic Considerations about Routing in $SCC_n$

Routing between two nodes  $\langle i_s, \pi_s \rangle$  and  $\langle i_d, \pi_d \rangle$  in  $SCC_n$  is equivalent to routing from  $\langle i_s, \pi_{ds} \rangle$  to  $\langle i_d, \pi_1 \rangle$ , where  $\pi_{ds} = \pi_d^{-1} \cdot \pi_s$ ,  $\pi_1 = 123 \dots n$ , and  $\pi_d^{-1}$  is the *inverse or reciprocal* of permutation  $\pi_d$ , such that  $\pi_d \cdot \pi_d^{-1} = \pi_1$  (Akers and Krishnamurthy, 1989; Latifi et al., 1993).

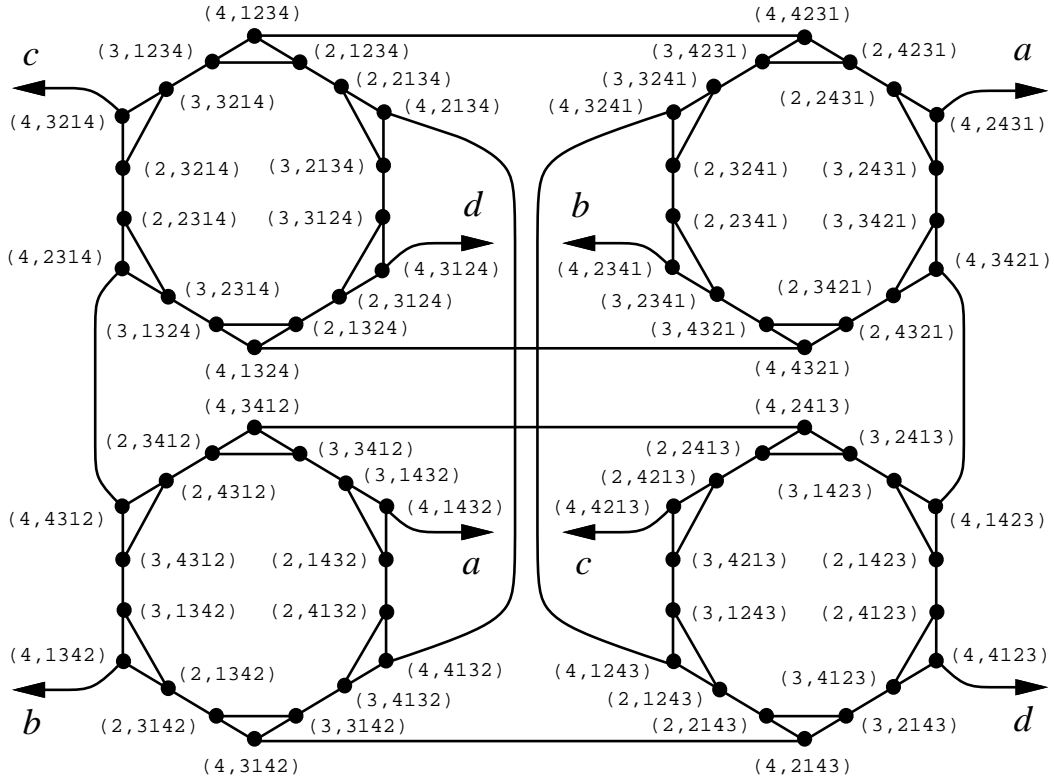


Fig. 1: The  $SCC_4$  graph

A routing algorithm that finds the shortest path between two nodes  $\langle i_s, \pi_s \rangle$  and  $\langle i_d, \pi_d \rangle$  in  $SCC_n$  is described in (Latifi et al., 1995). Such an algorithm computes a path  $P(l_1 \mapsto l_f)$ , which alternates between intracycle and intercycle routing (these are respectively accomplished by local and lateral links). The ordered sequence of lateral links within  $P(l_1 \mapsto l_f)$  is denoted by  $R(l_1 \mapsto l_f) = (l_1, l_2, \dots, l_f)$ . Hence, starting at supernode  $\pi_s$ ,  $P(l_1 \mapsto l_f)$  visits supernodes  $\pi_s \cdot l_1, \pi_s \cdot l_1 \cdot l_2, \dots, \pi_d$ .  $P(l_1 \mapsto l_f)$  has a cost of:

$$|P(l_1 \mapsto l_f)| = f + d(i_s, l_1) + \sum_{j=1}^{f-1} d(l_j, l_{j+1}) + d(l_f, i_d), \quad (3)$$

where  $d(i, j) = \min(|i - j|, n - 1 - |i - j|)$  denotes the number of local links that are traversed between the  $i^{th}$  and the  $j^{th}$  nodes of a supernode.

### EMBEDDING AN $n$ -DIMENSIONAL MESH INTO $SCC_n$

In this section, we consider the embedding of an  $n$ -dimensional mesh ( $M_n$ ) into  $SCC_n$ . Later, we show how the results of this section can be adapted to produce embeddings of meshes having fewer than  $n$  dimensions.

#### Mapping Technique

$M_n$  is defined to be an  $n$ -dimensional mesh of size  $|V(M_n)| = 2 \times 3 \times 4 \times \dots \times (n-1) \times n \times (n-1)$ . The nodes of  $M_n$  are labeled with an  $n$ -integer vector  $\omega = m_1 m_2 \dots m_n$ ,  $0 \leq m_j < s_j$ , where  $s_j$  is the width of  $M_n$  along its  $j^{th}$  dimension:

$$s_j = \begin{cases} j + 1, & \text{for } 1 \leq j \leq n - 1 \\ j - 1, & \text{for } j = n \end{cases} \quad (4)$$

Two nodes  $\omega = m_1 m_2 \dots m_n$  and  $\omega' = m'_1 m'_2 \dots m'_n \in V(M_n)$  are adjacent along the  $j^{\text{th}}$  dimension of  $M_n$  iff  $m_k = m'_k, \forall k \neq j$ , and either  $m_j = m'_j + 1$  or  $m_j = m'_j - 1$ . We assume that  $M_n$  does not have wrap-arounds.

The embedding of  $M_n$  into  $SCC_n$  is accomplished by Alg. 1, which maps a mesh node  $\omega = m_1 m_2 \dots m_n$  onto a image SCC node  $\langle i, \pi \rangle = \langle i, p_1 p_2 \dots p_n \rangle$ . Our algorithm is inspired by that proposed by Ranka et al. for embedding an  $(n - 1)$ -dimensional mesh into  $S_n$  (Ranka et al., 1993).

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mesh_to_SCC (int n, int m[ ], int i, int p[ ]) {
    int j, k, temp;
    for (j = 1; j ≤ n; j = j + 1) p[j] = j;
    for (j = 1; j < n; j = j + 1)
        for (k = 0; k < m[j]; k = k + 1) {
            temp = p[j - k];
            p[j - k] = p[j - k + 1];
            p[j - k + 1] = temp; }
    i = m[n] + 2; }
    
```

Alg. 1: Mapping of  $\omega \in V(M_n)$  onto  $\langle i, \pi \rangle \in V(SCC_n)$

Algorithm 1 stores  $\omega$  and  $\pi$  in vectors  $m[ ]$  and  $p[ ]$ , respectively. After initially setting  $\pi$  to  $12 \dots n$ , Alg. 1 processes the first  $(n - 1)$  coordinates of  $\omega$  as follows. Let the transposition of the symbols occupying the  $r^{\text{th}}$  and the  $s^{\text{th}}$  positions in  $\pi$  be denoted in cyclic format by  $(r\ s)$ . Table 1 lists the transpositions used by Alg. 1 along the first  $(n - 1)$  dimensions of the mesh. Note that if the  $j^{\text{th}}$  coordinate of  $\omega$  is  $m_j$ , then the  $j^{\text{th}}$  iteration of the externally nested *for* loop selects only the first  $m_j$  transpositions of the sequence corresponding to dimension  $j$  (i.e.,  $(j\ (j + 1)), ((j - 1)\ j), \dots, ((j - m_j + 1)\ (j - m_j + 2))$ ). Finally, the  $n^{\text{th}}$  coordinate of  $\omega$  (i.e.,  $m_n$ ) is directly mapped onto  $i$  via the assignment  $i = m_n + 2$ . Figure 2 shows the embedding of  $M_4$  into  $SCC_4$ , which is produced by Alg. 1.

Table 1: Sequences of transpositions used by Alg. 1

<i>Dimension (j)</i>	<i>Sequence of transpositions</i>
1	(1 2)
2	(2 3) (1 2)
⋮	⋮
$n - 1$	$(n - 1\ n) (n - 2\ n - 1) \dots (1\ 2)$

Algorithm 1 embeds  $M_n$  into  $SCC_n$  with load 1 and expansion 1, which respectively ensures that: 1) the amount of processing load in the nodes of  $SCC_n$  is the same as in  $M_n$ , and 2) no nodes in  $SCC_n$  are left unused. The bounded-degree topology of the  $SCC_n$  graph makes it difficult to obtain embeddings with small dilation (in fact, the dilation of our embedding of  $M_n$  into  $SCC_n$  grows with  $n$ ). However, as discussed later, a small communication slowdown can still be achieved if the local links of  $SCC_n$  have high bandwidth in comparison to the lateral links.

### Dilation

Before we can derive an expression for the dilation, a few definitions and lemmas are necessary. A path  $P(\ell_1 \mapsto \ell_f)$  in  $SCC_n$  is *normal* if it uses a sequence of lateral links  $R(\ell_1 \mapsto \ell_f) = (\ell_1, \ell_2, \dots, \ell_f)$ , such that  $d(\ell_k, \ell_{k+1}) = 1, \forall k, 1 \leq k < f$ . Note that Eq. 3 reduces to  $|P(\ell_1 \mapsto \ell_f)| = 2f - 1 + d(i_s, \ell_1) + d(\ell_f, i_d)$  if  $P(\ell_1 \mapsto \ell_f)$  is normal.

Normal paths use a minimum number of local links. Unfortunately, the existence of a normal path

between two arbitrary nodes  $\langle i_s, \pi_s \rangle$  and  $\langle i_d, \pi_d \rangle$  is not guaranteed. However, normal paths exist between the images of any two nodes  $\omega, \omega'$  which are adjacent along the  $(n-1)^{th}$  dimension of  $M_n$ .

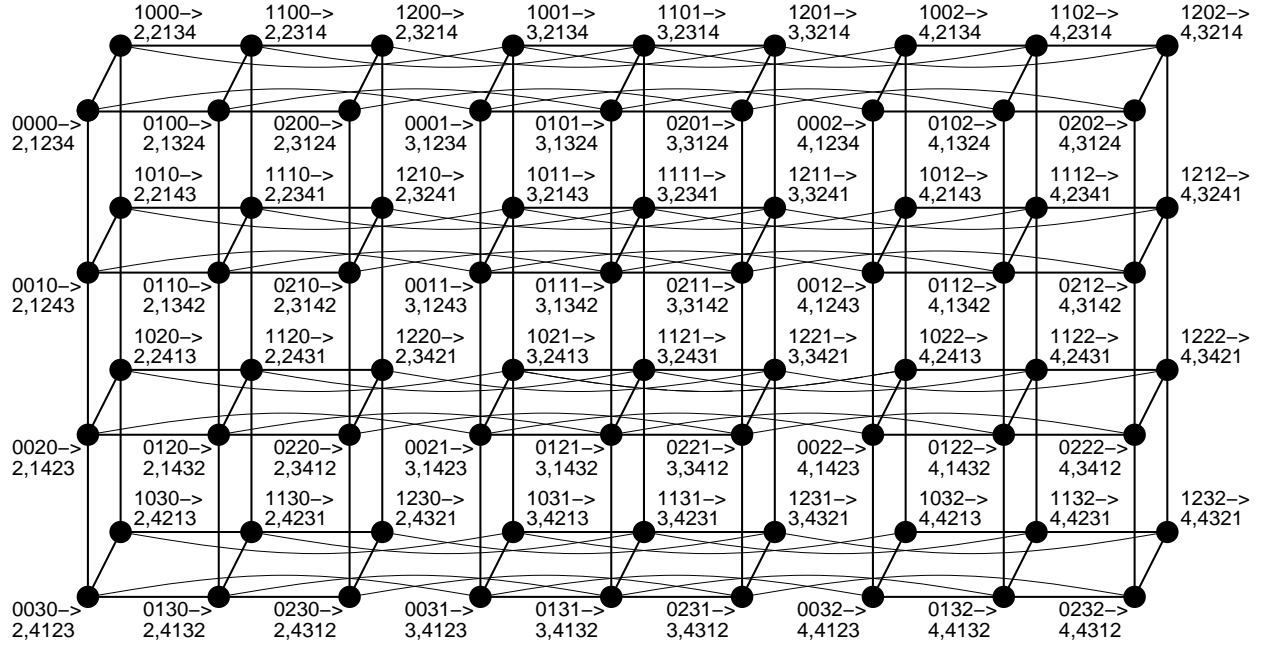


Fig. 2: Embedding of  $M_4$  into  $SCC_4$

The following three lemmas give the distances between the images of two nodes  $\omega, \omega' \in V(M_n)$ , respectively when: 1)  $\omega, \omega'$  are adjacent along the  $n^{th}$  dimension of  $M_n$ , 2)  $\omega, \omega'$  are adjacent along the  $(n-1)^{th}$  dimension of  $M_n$ , and 3)  $\omega, \omega'$  are adjacent along the  $j^{th}$  dimension of  $M_n$ ,  $j < n-1$ . Proofs for these lemmas are given in (Azevedo et al., 1995), and are not presented here due to space limitations.

**Lemma 1** Let two nodes  $\omega, \omega' \in V(M_n)$  be adjacent along the  $n^{th}$  dimension of  $M_n$ , and let the images of such nodes, as computed by Alg. 1, be respectively  $\langle i, \pi \rangle$  and  $\langle i', \pi' \rangle \in V(SCC_n)$ .  $\langle i, \pi \rangle$  and  $\langle i', \pi' \rangle$  are connected by a single local link in  $SCC_n$ .

**Lemma 2** Let two nodes  $\omega, \omega' \in V(M_n)$  be adjacent along the  $(n-1)^{th}$  dimension of  $M_n$ , and let the images of such nodes, as computed by Alg. 1, be respectively  $\langle i, \pi \rangle$  and  $\langle i', \pi' \rangle \in V(SCC_n)$ .  $\langle i, \pi \rangle$  and  $\langle i', \pi' \rangle$  are connected by a normal path of cost at most  $3 + 2\lfloor n/2 \rfloor$ , which contains a sequence of either 1 or 3 lateral links.

**Lemma 3** Let two nodes  $\omega, \omega' \in V(M_n)$  be adjacent along the  $j^{th}$  dimension of  $M_n$  ( $j < n-1$ ), and let the images of such nodes, as computed by Alg. 1, be respectively  $\langle i, \pi \rangle$  and  $\langle i', \pi' \rangle \in V(SCC_n)$ .  $\langle i, \pi \rangle$  and  $\langle i', \pi' \rangle$  are connected by a (not necessarily normal) path of cost at most  $3 + 2\lfloor (n-1)/2 \rfloor + 2\lfloor n/4 \rfloor$ , which contains a sequence of either 1 or 3 lateral links.

It becomes apparent from Lemmas 1, 2, and 3 that Alg. 1 produces a *variable-dilation* embedding of  $M_n$  into  $SCC_n$ , which is characterized by the following theorem:

**Theorem 1** Algorithm 1 produces a variable-dilation embedding  $F : M_n \mapsto SCC_n$ , which has load  $\lambda(F) = 1$ , expansion  $X(F) = 1$ , dilation  $d_j(F)$  along the  $j^{th}$  dimension of  $M_n$ , average dilation  $d_{avr}(F)$ , and dilation  $d(F)$ , where:

$$d_j(F) = \begin{cases} 3 + 2 \left\lfloor \frac{n-1}{2} \right\rfloor + 2 \left\lfloor \frac{n}{4} \right\rfloor, & \text{if } 1 \leq j < (n-1) \\ 3 + 2 \left\lfloor \frac{n}{2} \right\rfloor, & \text{if } j = n-1 \\ 1, & \text{if } j = n, \end{cases}$$

$$d(F) = 3 + 2 \left\lfloor \frac{n-1}{2} \right\rfloor + 2 \left\lfloor \frac{n}{4} \right\rfloor, \quad \text{and} \quad d_{avr}(F) = \frac{1}{n} \sum_{j=1}^n d_j(F) \quad (5)$$

Table 2 lists the dilation vector and the average dilation of our embedding of  $M_n$  into  $SCC_n$ , for  $3 \leq n \leq 7$ . Note that, among the embeddings depicted in Table 2, a smaller dilation is obtained along the  $(n-1)^{th}$ -dimension of  $M_n$  than along dimensions 1 through  $(n-2)$ , in the cases  $n=5$  and  $n=7$ . This result is due to the use of normal sequences of lateral links (see Lemma 2).

Table 2: Dilation vector and average dilation of embeddings of  $M_n$  into  $SCC_n$

$n$	$M_n$	$ V(M_n) $	Dilation vector ( $\overline{d(F)}$ )	Average dilation ( $d_{avr}(F)$ )
3	$2 \times 3 \times 2$	12	[5, 5, 1]	3.67
4	$2 \times 3 \times 4 \times 3$	72	[7, 7, 7, 1]	5.50
5	$2 \times 3 \times 4 \times 5 \times 4$	480	[9, 9, 9, 7, 1]	7.00
6	$2 \times 3 \times 4 \times 5 \times 6 \times 5$	3600	[9, 9, 9, 9, 9, 1]	7.67
7	$2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 6$	30240	[11, 11, 11, 11, 11, 9, 1]	9.29

#### EMBEDDING $p$ -DIMENSIONAL MESHES INTO $SCC_n$ ( $1 \leq p < n$ )

In what follows, we discuss how one can extend the technique described in the previous section to produce embeddings of  $p$ -dimensional meshes into  $SCC_n$ ,  $1 \leq p < n$ .

We initially consider the case  $p = n - 1$ . Let  $M_{n-1}^{j,k}$  be the  $(n-1)$ -dimensional mesh obtained by unfolding  $M_n$  in snakelike order along dimensions  $j$  and  $k$ .  $j$  and  $k$  are selected to produce a single dimension  $jk$  in  $M_{n-1}^{j,k}$ , such that the width of  $M_{n-1}^{j,k}$  along  $jk$  is  $s_j \times s_k$ .  $M_{n-1}^{j,k}$  has  $(n-1) \cdot n!$  nodes, and can be embedded with load 1 and expansion 1 into  $SCC_n$  as follows. We first transform the node labels of  $M_{n-1}^{j,k}$  into node labels of  $M_n$ , according to the order chosen to unfold  $M_n$ . Once that has been accomplished, Alg. 1 is used to produce an embedding whose dilation along dimension  $jk$  of  $M_{n-1}^{j,k}$  is  $\max(d_j, d_k)$ . This is justified by the fact that nodes that are adjacent in  $M_{n-1}^{j,k}$  along dimension  $jk$  were originally adjacent in  $M_n$  along either dimension  $j$  or dimension  $k$ . The dilation along any dimension  $\ell$  of  $M_{n-1}^{j,k}$ ,  $\ell \neq jk$ , is as given by Theor. 1.

This reasoning can be immediately extended to the case  $p < n - 1$ . For example, an  $(n-2)$ -dimensional mesh can be obtained by unfolding  $M_{n-1}^{j,k}$  along any two dimensions, and so on. An example which depicts the embedding of  $M_3^{1,2}$  (i.e., a mesh of size  $6 \times 4 \times 3$ ) into  $SCC_4$  is given in (Azevedo et al., 1995).

#### COMMUNICATION SLOWDOWN

In this section, we compare embeddings of  $n$ -dimensional meshes into SCC and star graphs. We estimate the communication slowdown of each particular embedding by taking into account its dilation and the bandwidth of the links of the host graph.

We note that our comparison evaluates embeddings which employ graphs of *similar (but not equal) size*. Namely, we compare: 1) our embedding of  $M_n$  into  $SCC_n$ , where  $|V(M_n)| = |V(SCC_n)| = (n-1) \cdot n!$ , and 2) the embedding of  $\widehat{M}_n$  into  $S_{n+1}$ , where  $\widehat{M}_n$  is an  $n$ -dimensional mesh of size  $|V(\widehat{M}_n)| = |V(S_{n+1})| = 2 \times 3 \times \cdots \times (n+1) = (n+1)!$ . We denote these embeddings respectively by  $F$  and  $\widehat{F}$ .  $\widehat{F}$  is described in (Ranka et al., 1993), and has load 1, expansion 1, and constant dilation 3.

As explained in (Latifi et al., 1995), SCC graphs can use high-bandwidth parallel buses in the local links, at the expense of a minimum increase in the layout complexity. The layout of the lateral links, however, is characterized by similar wiring constraints found in high-dimensional networks such as the star graph and the hypercube. Hence, lateral links must often be implemented with narrower-bandwidth communication links.

Because local links have a dominant contribution to  $d_j(F)$  when  $1 \leq j < n$  (see Eq. 5, Lemma 2, and Lemma 3),  $F$ 's communication slowdown is largely minimized if high-bandwidth links are used within the supernodes. We denote the bandwidth of the local and lateral links of an SCC graph by  $B_{loc}(SCC_n)$  and  $B_{lat}(SCC_n)$ , respectively, and define  $\nu = B_{loc}(SCC_n)/B_{lat}(SCC_n)$ . Although the links of  $S_{n+1}$  are constrained by a more complex layout than the lateral links of  $SCC_n$ , we assume in our comparison that the bandwidth of the links of  $S_{n+1}$  is also  $B_{lat}(SCC_n)$ .

An evaluation of the actual communication slowdown produced by either  $F$  or  $\widehat{F}$  must take into account the bandwidth of the links in  $M_n$  and  $\widehat{M}_n$ , respectively. Because our goal is to compare host graphs, it suffices to select a reference bandwidth for the links of  $M_n$  and  $\widehat{M}_n$  (we choose  $B_{lat}(SCC_n)$ ). We denote  $\widehat{F}$ 's communication slowdown by  $\eta(\widehat{F})$ , and use the estimate  $\eta(\widehat{F}) = d(\widehat{F}) = 3$ . Because  $F$  is a variable-dilation embedding, we define  $\eta_j(F)$  and  $\eta_{avr}(F)$  to be respectively: 1) the communication slowdown produced by  $F$  along the  $j^{th}$  dimension of  $M_n$ , and 2) the average communication slowdown produced by  $F$ . Given our previous assumptions,  $\eta_j(F)$  and  $\eta_{avr}(F)$  can be estimated by (Azevedo et al., 1995):

$$\eta_j(F) = \begin{cases} 3 + \frac{2}{\nu} \left\lfloor \frac{n-1}{2} \right\rfloor + \frac{2}{\nu} \left\lfloor \frac{n}{4} \right\rfloor, & \text{if } 1 \leq j < (n-1) \\ 3 + \frac{2}{\nu} \left\lfloor \frac{n}{2} \right\rfloor, & \text{if } j = n-1 \\ \frac{1}{\nu}, & \text{if } j = n \end{cases} \quad \text{and} \quad \eta_{avr}(F) = \frac{1}{n} \sum_{j=1}^n \eta_j(F) \quad (6)$$

Table 3 compares estimates for the communication slowdown produced by embeddings of  $n$ -dimensional meshes into  $S_{n+1}$  and  $SCC_n$ . Values for  $\eta_{avr}(F)$  are computed from Eq. 6, assuming that  $\nu = n$ . Such an assumption tries to reflect the fact that, as  $n$  grows, the size of both  $S_{n+1}$  and  $SCC_n$  increases rapidly, augmenting the layout complexity of the lateral links. As a consequence, the ratio  $\nu$  is expected to increase as well. A more precise investigation on the dependence of  $\nu$  with  $n$  lies outside the scope of this paper, and is the object of future research combining the VLSI layout and link bandwidths of the SCC graph. In fact, we suspect that  $\nu$  actually grows at an even faster rate than the assumption  $\nu = n$  adopted in this paper.

Table 3: Communication slowdown of embeddings of  $n$ -dimensional meshes into  $S_{n+1}$  and  $SCC_n$

$n$	$\widehat{F}$	$F$	$ V(S_{n+1}) $	$ V(SCC_n) $	$\eta(\widehat{F})$	$\eta_{avr}(F)$
3	$\widehat{M}_3$ into $S_4$	$M_3$ into $SCC_3$	24	12	3	2.56
4	$\widehat{M}_4$ into $S_5$	$M_4$ into $SCC_4$	120	72	3	3.06
5	$\widehat{M}_5$ into $S_6$	$M_5$ into $SCC_5$	720	480	3	3.32
6	$\widehat{M}_6$ into $S_7$	$M_6$ into $SCC_6$	5040	3600	3	3.36
7	$\widehat{M}_7$ into $S_8$	$M_7$ into $SCC_7$	40320	30240	3	3.53

Table 3 indicates that  $SCC_n$  can be an attractive host for mesh embeddings, which is not immediately evident from the dilation figures shown in Table 2. As depicted in Table 3,  $SCC_n$  and  $S_{n+1}$  can in fact present similar performance as far as mesh embeddings are concerned. Other interesting criteria are used to compare SCC and star graphs in (Latifi et al., 1993; Latifi et al., 1995).

## CONCLUSION

This paper presented an embedding of an  $n$ -dimensional mesh ( $M_n$ ) into an  $SCC_n$  graph, which has load 1, expansion 1, and variable dilation (the dilation varies from 1 to  $3 + 2\lfloor(n-1)/2\rfloor + 2\lfloor n/4\rfloor$ , according to the dimensions of the mesh). Our results indicate that both the dilation  $d(F)$  and the average dilation  $d_{avr}(F)$  of our embedding grow with  $n$ . From  $n = 3$  to  $n = 7$ , for example,  $d(F)$  grows from 5 to 11, while  $d_{avr}(F)$  grows from 3.67 to 9.29. The dependency of  $d(F)$  and  $d_{avr}(F)$  on  $n$  is caused by the bounded-degree topology of the SCC graph and its routing characteristics. In contrast,  $n$ -dimensional meshes can be embedded into  $S_{n+1}$  with constant dilation 3 (Ranka et al., 1993).

The dilation  $d(F)$  of our embedding is derived from a worst-case analysis, which reveals paths containing at most 3 lateral links and  $2\lfloor(n-1)/2\rfloor + 2\lfloor n/4\rfloor$  local links. Hence, lateral links have a fixed contribution to the dilation of the embedding, making our approach similar to that of (Ranka et al., 1993) in that respect. Noting that the topology of the SCC graph favors implementations using high-bandwidth buses in the local links, we showed that SCC and star graphs can achieve similar communication slowdowns when used as hosts for mesh embeddings.

An extension of our embedding of  $M_n$  into  $SCC_n$  was also presented in this paper, which addressed the more general case of embeddings of  $p$ -dimensional meshes into  $SCC_n$ , for  $1 \leq p < n$ .

## REFERENCES

- Akers, S. B., D. Harel and B. Krishnamurthy (1987). The star graph: an attractive alternative to the  $n$ -cube. *Proc. Int'l Conf. on Parallel Processing*, 393-400.
- Akers, S. B. and B. Krishnamurthy (1989). A group-theoretic model for symmetric interconnection networks. *IEEE Transactions on Computers*, Vol. 38, No. 4, 555-566.
- Azevedo, M. M., N. Bagherzadeh and S. Latifi (1995). *Embedding meshes into the star-connected cycles interconnection network*. Technical Report ECE 95-10-01. Department of Electrical and Computer Engineering, University of California, Irvine.
- Latifi, S., M. M. Azevedo and N. Bagherzadeh (1993). The star-connected cycles: a fixed-degree interconnection network for parallel processing. *Proc. Int'l. Conf. Parallel Processing*, Vol. 1, 91-95.
- Latifi, S., M. M. Azevedo and N. Bagherzadeh (1995). A star-based I/O-bounded network for massively parallel systems. *IEE Proceedings - Computers and Digital Techniques*, Vol. 142, No. 1, 5-14.
- Leighton, F. T. (1992). *Introduction to Parallel Algorithms and Architectures: Arrays · Trees · Hypercubes*. Morgan Kaufmann Publishers, San Mateo, California.
- Ranka, S., J.-C. Wang and N. Yeh (1993). Embedding meshes on the star graph. *Journal of Parallel and Distributed Computing*, 19, 131-135.