Chapter 2

Amplification

2.1 Linear Amplifier Representations

Power amplification is an essential function of analog circuits. In general an amplifier is presented as a two-port, shown in Fig. 2.1(a), with one port assigned as the input and the other as the output. Regardless of what is actually inside the network in Fig. 2.1(a), the amplifier can often be modeled, at low frequencies, as a simple configuration such as the one shown in Fig. 2.1(b). The amplification itself is modeled by a dependent source – in this case, a voltage-controlled voltage source (VCVS) is shown. Input resistance $r_{in}$ models the interaction between the output of the preceding stage and the amplifier’s input circuitry. Likewise, resistance $r_{out}$ models the interaction between the input of the next stage and the amplifier’s output circuitry. As will be seen shortly, all three parameters $A_v$, $r_{in}$ and $r_{out}$ play

![Voltage amplifier block diagram](image)

Figure 2.1: Voltage amplifier block diagram.
important roles in an amplifier’s ability to provide power amplification.

The input and output resistances are always modeled in any amplifier representation; however, the nature of the input signal (i.e., choice of voltage or current) and the type of dependent source can be chosen based on convenience and ease of analysis. In Fig. 2.2 is shown all four of the possible “hybrid” representations. Which one of these four hybrid representations is preferred for a given amplifier circuit is usually determined by the input and output resistances. For large (small) $r_{in}$, it is more convenient to choose the input to be in the form of a voltage (current); for small (large) $r_{out}$, it is more convenient to choose the output to be in the form of a voltage (current). The reason for this will be seen shortly when we analyze how multiple amplifier blocks behave when connected together.

In order to properly analyze a circuit made up of a number of blocks, it is often important to model each block in one of the forms represented in Fig. 2.1(b). This modeling comes from taking a set of “measurements”
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(this could be actual lab measurements, or it could be simulation or hand analysis) at the two ports of the block. We begin our study of this block diagram by determining the simplest way of measuring each parameter in Fig. 2.1(b).

1. To find the input resistance \( r_{in} \), we simply connect a voltage source \( v_{in} \) directly to the input and measure the current \( i_{in} \) that results, as illustrated in Fig. 2.3. However, it happens that in certain amplifiers the value of \( r_{in} \) depends on the load resistance \( R_L \) that is connected to the output. (This fact is not obvious from observing the Fig. 2.1 diagram, but it will be made clear later when we study various amplifier realizations.) Thus we must include \( R_L \) in the circuit to be analyzed, as shown in Fig. 2.3. It's clear from this circuit that \( r_{in} = v_{in} / i_{in} \).

2. To find the output resistance \( r_{out} \), we first arrange to have the dependent source turned off (i.e., set to zero) by not connecting any independent source to the input. As illustrated in Fig. 2.4, we can directly measure \( r_{out} \) by setting \( v_{in} \) to 0 – that is, replacing it with a short circuit – and then applying a voltage source \( v_x \) to the output and measuring the resulting current \( i_x \). Similar to the expression for \( r_{in} \), in general \( r_{out} \) can for some configurations be a function of source resistance \( R_S \). Thus \( R_S \) needs to be connected to the input as illustrated in Fig. 2.4. For this circuit \( v_{in} \) must be zero (why?); thus it's clear that \( r_{out} = \frac{v_x}{i_x} \). (Note that we could just as easily have connected a current source to the output and then determine corresponding output voltage. The resulting hand analysis and result would be identical either way.)

3. To find the voltage gain \( A_v \) in the most direct way, we connect a voltage source \( v_{in} \) directly to the input and leave the output open-circuited as shown in Fig. 2.5. It's clear from this figure that \( A_v = \frac{v_{out}}{v_{in}} \), independent of \( r_{in} \) and \( r_{out} \). (The procedure for finding the other dependent source coefficients in the Fig. 2.2(b), (c) and (d) representations is left as a homework exercise.)

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It is clear that the voltage (or current) gain of the dependent source is of great interest to the amplifier designer since it is this element that provides the actual amplification. However, at this point it is probably less clear why
Figure 2.3: Circuit for measuring input resistance \( r_{in} \).

Figure 2.4: Circuit for measuring output resistance \( r_{out} \).

Figure 2.5: Circuit for measuring voltage gain \( A_v \).
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![Active Circuit Diagram](image)

Figure 2.6: Active circuit.

$r_{in}$ and $r_{out}$ are of interest. To gain some insight into the importance of these resistances, we consider the active circuit shown in Fig. 2.6, the same circuit whose power gain was analyzed in Section 1.4. As discussed in that section, the power gain was found to be approximately $A^2 r_{in} / R_L$. Notice that even if $A$ is less than unity (i.e., voltage is not amplified), power amplification can still occur, depending on the internal resistances – i.e., if $r_{in}$ is sufficiently high and $r_{out}$ is sufficiently low. Later in this chapter we will see examples of useful circuits that have voltage or current gain close to 1.

It is usually the case that the external resistances $R_S$ and $R_L$ are fixed, and the designer must find an appropriate amplifier, subject to these constraints, that gives the desired power amplification.

### 2.3 One-Transistor Amplifiers

We will now study the most simple class of amplifiers: those that can be realized with just one transistor. As mentioned earlier, an amplifier is defined as a two-port as illustrated in Fig. 2.7(a). Moreover, it is common to consider a more restricted class of 2-ports where the input and output share a common terminal, as illustrated in Fig. 2.7(b), since it is usually convenient to define the input and output with respect to the same reference – the small-signal ground.

Sketches of six different one-transistor configurations are shown in Fig. 2.8. In particular, Fig. 2.8(a), (b) and (c) show all three of the possible common two-port configurations for a single BJT: Fig. 2.8(a) shows the configuration with the emitter chosen as the common terminal; Fig. 2.8(b) with the collector as the common terminal; Fig. 2.8(c) with the base as the common terminal. Likewise, Fig. 2.8(d), (e) and (f) show the corresponding configurations (common source, common drain, and common gate, respectively).
for a single MOSFET.

Although the circuits shown in Fig. 2.8 represent all of the possible common two-port configurations, the selection of which port is input and which is output is not unique. The assignments of the input and output for each Fig. 2.8 representation are based on desirable amplifier functionality and range of input and output resistances. This will become more clear as we analyze each configuration in detail. In addition, we emphasize that these are only sketches of the actual circuit realizations. As we will show shortly, additional circuitry for biasing (including independent sources and resistors) is necessary for proper functioning.

2.3.1 Common-Emitter/Common-Source Amplifier

Corresponding to the sketch in Fig. 2.8(a), where the emitter terminal is common to both ports, a common-emitter amplifier is shown in Fig. 2.9(a). To bias this circuit in the forward-active region, a dc voltage source $V_{BE}$ is applied between the base and emitter of the transistor; its value is set to realize a certain dc collector current $I_C$. (Note that $I_C$ varies exponentially with $V_{BE}$; in practice it would be extremely difficult to set this voltage directly with enough precision to realize the desired current. This issue will be discussed later.) A small-signal input $v_{in}$ is applied in series with $V_{BE}$ and the small-signal output $v_{out}$ is taken between the collector and emitter. (Just how “small” we mean when we talk about small-signal inputs and outputs will be discussed shortly.) A resistor $R_C$, connected between $V_{CC}$ and the collector, makes $V_{out}$ dependent on the collector current and therefore on $v_{in}$. 

Figure 2.7: (a) General 2-port representation; (b) special case of common 2-port.
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Figure 2.8: One-transistor amplifier sketches represented as common two-ports.

Figure 2.9: (a) Common-emitter BJT amplifier; (b) common-source MOS-FET amplifier.
Figure 2.10: Voltage transfer characteristics of the Fig. 2.9(a) and (b) amplifiers.

Common-Emitter Large-Signal Characteristic

Before performing any calculations we first construct, based on qualitative analysis, an approximation of the dc voltage transfer characteristic of the common-emitter amplifier. For $V_{BE} < V_{BE(on)}$ the transistor is biased in the cutoff region and the collector current is nearly zero; thus $V_{OUT} \approx V_{CC}$. As $V_{BE}$ approaches $V_{BE(on)}$, the transistor begins to turn on, which results in the collector current $I_C$ increasing exponentially with $V_{BE}$. For this narrow range of $V_{BE}$, the transistor is biased in the forward active region and as a result, since $V_{OUT} = V_{CC} - I_C R_C$, $V_{OUT}$ decreases exponentially. Once $V_{OUT}$ reaches $V_{CE(sat)}$, the transistor enters the saturation region, and $V_{OUT}$ suddenly stops decreasing, remaining constant at $V_{CE(sat)}$. These three operating regions are illustrated in the voltage transfer characteristic in Fig. 2.10(a). Since our intent is to design an amplifier with large voltage gain, we need to bias the transistor within the forward active region, since that region has the largest magnitude slope of $V_{OUT}$ vs. $V_{BE}$.

The region of the curve for which $V_{BE}$ is more than approximately 100 mV higher than $V_{BE(on)}$ is shown as a dashed line because a BJT is normally not operated in this region. Although the collector current is constrained to a reasonable value, since both p-n junctions sustain high voltages, the base and emitter currents can become very high, thus causing the transistor a
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dangerously high amount of current.

**Sensitivity of DC Bias Point**

To illustrate the nature of the exponential behavior of a BJT, we can perform the following calculation: Suppose we have selected nominal values of dc voltages \( V_{BE(nom)} \) and \( I_{C(nom)} \), which are related as follows:

\[
I_{C(nom)} = \alpha_F I_{ES} \cdot \left[ \exp \left( \frac{V_{BE(nom)}}{V_T} \right) - 1 \right] \quad (2.1)
\]

\[
\approx \alpha_F I_{ES} \cdot \exp \left( \frac{V_{BE(nom)}}{V_T} \right)
\]

Now let us increase the \( V_{BE} \) by a small voltage \( \Delta V \). The resulting collector current will be given by:

\[
I_C = \alpha_F I_{ES} \cdot \exp \left( \frac{V_{BE(nom)} + \Delta V}{V_T} \right)
\]

\[
= \alpha_F I_{ES} \cdot \exp \left( \frac{V_{BE(nom)}}{V_T} \right) \cdot \exp \left( \frac{\Delta V}{V_T} \right)
\]

\[
= I_{C(nom)} \cdot \exp \left( \frac{\Delta V}{V_T} \right) \quad (2.2)
\]

From (2.2) we can conclude that, at room temperature (corresponding to \( V_T = 26 \text{ mV} \)), an increase of just 60 mV in \( V_{BE} \) results in \( I_C \) increase by more than a factor of 10! Because of this high sensitivity (which in fact is essential to realizing high voltage gain), biasing of this amplifier using a dc voltage source connected between the base and emitter is in practice very difficult and subject to large variations. More robust biasing techniques for amplifiers such as this one will be discussed in detail in Chapter 3.

**Large-Signal vs. Small-Signal Behavior**

To understand the distinction between small-signal and large-signal operation, suppose that we now apply a small-signal input voltage \( v_{in} \) placed in series with dc voltage source \( V_{BE} \). This is illustrated in Fig. 2.11(a), where the small-signal input voltage, shown on the horizontal axis, is assumed to be sinusoidal. The resulting output signal, shown on the vertical axis, exhibits a much higher amplitude than the input, as expected. However, it is also apparent that the output signal is distorted; it differs from a pure sinusoidal signal in that the lower portion of the waveform is “stretched” vertically more than the upper portion. This is due to the nonlinear nature
Figure 2.11: Illustration of effect of nonlinearity in small-signal amplification.

of the voltage transfer characteristic. This effect becomes even more pronounced when we double the input amplitude as illustrated in Fig. 2.11(b). For this case, the output is seen to flatten out near the extreme high and low voltages, thus exhibiting even more distortion.

In general, we say that an amplifier exhibits “small-signal” behavior as long as the output signal exhibits sufficiently low distortion – where “sufficiently low” would be defined according to the particular application of the circuit. In most cases, the lower the input amplitude, the more linear the circuit behavior, and thus the closer to sinusoidal the output. A small-signal analysis by definition assumes that the input signal amplitude is small enough such that the output is a purely linear function of the input.

Common-Source Amplifier Analysis

We can perform a similar qualitative analysis on the Fig. 2.9(b) commonsource MOSFET amplifier, which is based on the Fig. 2.8(d) sketch. The voltage transfer characteristic of this amplifier is shown in Fig. 2.10(b). For $V_{GS} < V_t$ the transistor is in the cutoff region and the drain current is nearly zero; thus $V_{OUT} \approx V_{DD}$. As $V_{GS}$ increases past $V_t$, the transistor begins
to turn on, which results in the drain current $I_D$ increasing quadratically with $V_{GS}$. The transistor is then biased in the saturation region and as a result, $V_{OUT}$ decreases quadratically as shown in Fig. 2.10(b). Also shown in this figure is a dashed line corresponding to $V_{OUT} = V_{GS} - V_t$. This line defines the boundary between the MOSFET’s saturation and triode regions of operation. The point where this line intersects the voltage transfer characteristic is known as the “edge of saturation.” The gate-to-source voltage at this point is labeled $V_{GS(qos)}$ and the corresponding drain voltage is labeled $V_{DSAT}$. Once $V_{OUT}$ reaches $V_{DSAT}$ the transistor enters the triode region and $V_{OUT}$ then flattens out. Unlike the BJT case where $V_{CE(sat)}$ is nearly constant, the value of $V_{DSAT}$ is dependent on the specific transistor parameters and other component values. Analyzing this region is difficult to do exactly, but a qualitative understanding of the shape of the curve can be found by observing the curves shown in Fig. 2.12(a). Corresponding to the Fig. 2.9(b) common-source amplifier, the bold lines are a family of curves, for different values of $V_{GS} - V_t$, that describe the $I_D$ vs. $V_{DS}$ MOSFET characteristic. The straight line describes the load line for the resistor. For a given value of $V_{GS} - V_t$ the intersection between the two lines gives the circuit’s operating point. It can be seen by inspection from these curves that $V_{DS}$ decreases monotonically toward zero as $V_{GS}$ increases. A similar analysis on the Fig. 2.12(b) curves, corresponding to the Fig. 2.9(a) common-emitter amplifier, illustrates further how this amplifier exhibits a “hard” limit of $V_{CE(sat)}$ when the BJT becomes saturated.
Figure 2.13: Small-signal circuit schematic for: (a) common-emitter amplifier; (b) common-source amplifier.

Small-Signal Parameters of Common-Emitter & Common-Source Amplifiers

As described in Section 2.1, it is desirable to model an amplifier’s small-signal behavior as the simple 2-port shown in Fig. 2.1(b). Thus we will analyze the Fig. 2.9(a) and (b) circuits to derive their equivalent $A_v$, $r_{in}$, and $r_{out}$. The small-signal schematics for the common-emitter and common-source amplifiers are shown in Fig. 2.13(a) and (b), respectively.

Voltage Gain

Using the Fig. 2.5 schematic as a guide, we can derive the small-signal parameters of the common-emitter amplifier as shown in Fig. 2.13(a) for use in finding voltage gain $A_v$.

Noting that resistors $r_o$ and $R_C$ are connected in parallel in this circuit, we can write a KCL equation at node $v_{out}$:

$$\frac{v_{out}}{R_C|r_o} + g_m v_\pi = 0$$

(2.3)
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We can also express branch voltage $v_x$ in terms of the node voltages:

$$ v_x = v_{in} \quad (2.4) $$

After combining the above equations, we have the following expression for the gain $A_v$:

$$ A_v \equiv \frac{v_{out}}{v_{in}} = -g_m \left( R_C | r_o \right) \quad (2.5) $$

Note that the voltage gain for this configuration is independent of $r_x$, since for this analysis the small-signal input voltage is applied directly between the base and emitter.

Before proceeding further, let us consider in more detail the expression $R_C | r_o$, which is the parallel combination of $r_o$, a small-signal resistance modeling the Early effect of a BJT, and $R_C$, a “real” physical resistor. Using $r_o = V_A / I_C$, we can express the parallel resistance in terms of biasing parameters:

$$ R_C | r_o = \frac{1}{\frac{1}{R_C} + \frac{1}{r_o}} = \frac{1}{\frac{1}{R_C} + \frac{I_C}{V_A}} \quad (2.6) $$

$$ = \frac{R_C}{1 + \frac{I_C R_C}{V_A}} \quad (2.7) $$

The expression $I_C R_C / V_A$ found in the denominator of (2.7) is the ratio of two voltages: $V_A$ is the transistor Early voltage, typically 75 to 100 V for an npn transistor, while, referring to Fig. 2.9(a), $I_C R_C$ is the dc voltage drop that appears across resistor $R_C$. Since no node voltage in the circuit can be higher than $V_{CC}$, the voltage across $R_C$ is constrained to be less than $V_{CC}$, which is typically no more than a few volts. Thus it is normally the case that we can write:

$$ I_C R_C \ll V_A \quad (2.8) $$

and, applying this inequality to (2.7), we can conclude that $R_C | r_o \approx R_C$ and thus, from (2.5) $A_v \approx -g_m R_C$. We’ll find this approximation often, where $r_o$ is much bigger than an actual on-chip resistor.

We can also express $g_m$ in terms of biasing parameters:

$$ g_m = \frac{I_C}{V_T} \quad (2.9) $$

Combining (2.5), (2.7), and (2.9), we have the following expression for the gain in terms of biasing parameters:

$$ A_v = \frac{I_C}{V_T} \left[ \frac{R_C}{1 + \frac{I_C R_C}{V_A}} \right] $$
\begin{equation}
A_v \approx -\frac{I_C R_C}{V_T} \cdot \frac{1}{1 + L_{E_C}/V_A} \approx -\frac{I_C R_C}{V_T} \label{eq:2.10}
\end{equation}

**Practical Limitations on Voltage Gain**

By observing only the small-signal expression in (2.5) it would appear as though we could make the amplifier gain arbitrarily large simply by making certain parameters in those expressions (e.g., $g_m$ and/or resistance) sufficiently high. However, it turns out that the large-signal biasing of the amplifiers imposes a severe constraint on the maximum gain that can be realized by analyzing the expression in (2.10). As mentioned before, if we assume that, due to supply voltage limitations, $I_C R_C \ll V_A$, we can simplify (2.10) to:

\begin{equation}
A_v \approx -\frac{I_C R_C}{V_T} \label{eq:2.11}
\end{equation}

Note that this simplified expression is also the ratio of two voltages: The numerator is, once again, the dc voltage drop across resistor $R_C$, and the denominator is the thermal voltage $V_T$, which is constant at a given temperature (26 mV at room temperature.) As mentioned earlier, the maximum realizable voltage $I_C R_C$ is limited by the value of $V_{CC}$. However, there can be an even more strict constraint on this voltage. It is often desirable to arrange to have the maximum possible output voltage swing for which the transistor remains biased in the forward-active region. By observing the voltage transfer characteristic in Fig. 2.10(a) (and replicated in Fig. 2.14(a)), we see that in this region $V_{OUT}$ can range from a minimum of $V_{CE(sat)}$ to a maximum of $V_{CC}$. Thus the optimum dc output voltage $V_{OUT(opt)}$ would be the exact middle of this range, as illustrated in Fig. 2.14(a); i.e.,

\begin{equation}
V_{OUT(opt)} = \frac{1}{2} \left[ V_{CC} + V_{CE(sat)} \right] \label{eq:2.12}
\end{equation}

By using KVL, we can then determine the dc voltage drop across $R_C$ corresponding to the optimum operating point specified in (2.11):

\begin{equation}
I_C R_C = V_{CC} - V_{OUT(opt)} \label{eq:2.13}
\end{equation}

\begin{equation}
I_C R_C = \frac{1}{2} \left[ V_{CC} - V_{CE(sat)} \right] \label{eq:2.14}
\end{equation}

Substituting (2.13) back into (2.10), we have:

\begin{equation}
A_v \approx -\frac{V_{CC} - V_{CE(sat)}}{2V_T} \label{eq:2.15}
\end{equation}
2.3. ONE-TRANSISTOR AMPLIFIERS

Figure 2.14: Illustration of optimum biasing for maximum output swing.

Notice that this expression depends completely on the supply voltage and the physical constant $V_T$! In fact, the designer has very little freedom in setting $A_v$ for this circuit.
Example 3.1:
In the common-emitter amplifier shown in Fig. 2.9(a), let \( V_{CC} = 3.3 \text{V}, \) \( V_{CE(sat)} = 0.2 \text{V}, \) and \( V_T = 0.026 \text{V}. \) Plugging these values into (2.14), we obtain an inverting gain of about 60.

To verify this result, let us look at more details of this design:
We begin by choosing \( I_C = 500 \mu \text{A}. \) (For these early examples, we will assume for now that the designer can arbitrarily choose the biasing current. In reality, this choice is based on a number of considerations, such optimizing the transistor for maximum \( \beta \) and/or maximum speed, as well as minimizing the amplifier’s power dissipation and/or noise generation. These issues will be discussed more throughout this book.)

In order to achieve the optimum dc biasing – that is, setting \( V_{OUT} = \frac{1}{2}[V_{CC} + V_{CE(sat)}] = 1.75 \text{V}, \) we should have the following:

\[
I_C R_C = V_{CC} - V_{OUT} = 1.55 \text{ V} \tag{2.15}
\]

Using the value already chosen for \( I_C, \) we then have:

\[
R_C = 1.55 \text{V} / 500 \mu \text{A} = 3.1 \text{k}\text{\Omega} \tag{2.16}
\]

Let us suppose now, based on the transistor model, that the transistor Early voltage \( V_A = 100 \text{ V}. \) Then we can derive the following small-signal parameters:

\[
g_m = \frac{500 \mu \text{A}}{26 \text{ mV}} = 19.23 \text{ mS} \\
r_o = \frac{100 \text{ V}}{500 \mu \text{A}} = 200 \text{ k}\text{\Omega}
\]

Applying the above small-signal expressions into (2.5), we can finally write:

\[
A_v = -(19.23 \text{ mS}) \cdot (3.1 \text{ k}\text{\Omega} / 200 \text{ k}\text{\Omega}) \\
= -19.23 \text{ mS} \cdot (3.05 \text{ k}\text{\Omega}) \\
= -58.7
\]

Note that the above calculations give \( R_C || r_o = 3.05 \text{ k}\text{\Omega}, \) which is indeed very close to \( R_C = 3.1 \text{ k}\text{\Omega}, \) within 2\%, consistent with our earlier assumption.
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If we were to attempt to increase the gain by, for example, increasing $R_C$ while keeping $I_C$ (and therefore $g_m$) constant, the dc operating point would deviate from that specified in (2.11). Thus, in order to restore this optimum dc biasing, we would have to reduce $I_C$, which would then reduce the gain back to its original value.

Common-Source Amplifier

Turning our attention now to the small-signal schematic of the common-source amplifier shown in Fig. 2.13(b), we can analyze it the same way as the Fig. 2.13(a) circuit. The gain of this amplifier, whose expression is identical in form to (2.5), is:

$$A_v \equiv \frac{v_{out}}{v_{in}} = -g_m (R_D | r_o)$$  \hspace{1cm} (2.17)

As in the case for a BJT, we can write the parallel combination of $r_o$ and $R_D$ as:

$$R_D | r_o = \frac{1}{\frac{1}{r_o} + \lambda I_D}$$  \hspace{1cm} (2.18)

$$= \frac{R_D}{1 + \lambda I_D R_D}$$  \hspace{1cm} (2.19)

The value of $\lambda$ can take a wide range of values depending on the fabrication technology and on the chosen gate length. If, for example, $\lambda$ were 0.1 $V^{-1}$, then the term in the denominator in (2.19) would be given by:

$$\lambda I_D R_D = \frac{I_D R_D}{10}$$  \hspace{1cm} (2.20)

The expression in (2.20) is the ratio of the voltage drop across $R_D$, which is limited by the supply voltage $V_{DD}$, to 10V. Thus it is normally the case that $\lambda I_D R_D << 1$, although this inequality for a MOSFET is not as strong as the one in (2.8) for a BJT.

In order to convert (2.17) into an expression involving biasing parameters, we first write $g_m$ as:

$$g_m = \frac{2I_D}{V_{GS} - V_t}$$  \hspace{1cm} (2.21)

Substituting (2.21) and (2.19) into (2.17), we have:

$$A_v = -\frac{2I_D}{V_{GS} - V_t} \cdot \left[ \frac{R_D}{1 + \lambda I_D R_D} \right]$$  \hspace{1cm} (2.22)
If we assume $I_D R_D << 1/\lambda$, (2.22) can be simplified to

$$A_v \approx -\frac{2I_D R_D}{V_{GS} - V_t} \tag{2.23}$$

Once again we see that the gain is proportional to the dc drop in resistor $R_D$. For the Fig. 2.9(b) common-source amplifier, it is not quite as straightforward to determine the output voltage swing and optimum bias point as for the Fig. 2.9(a) common-emitter amplifier since there is no equivalent “hard” saturation level for the common-source amplifier. Referring to the Fig. 2.14(b) graph we will consider the maximum swing that corresponds to the MOSFET being in the saturation region of operation. As indicated in this graph, the point the marks the boundary between saturation and triode, known as “edge of saturation,” is defined by where $V_{DS} = V_{GS} - V_t$. At this point the drain-to-source voltage is often referred to at $V_{DSAT}$, and we will refer to the gate-to-source voltage at this point as $V_{GS(\text{opt})}$. Using an analysis similar to the one for the BJT swing, we can now define the optimum bias point for the common-source amplifier as the average between the maximum and minimum output voltages:

$$V_{OUT(\text{opt})} = \frac{1}{2} [V_{DD} + V_{DSAT}] \tag{2.24}$$

We can also write the expression for the voltage drop across $R_D$ at the bias point:

$$I_D R_D = V_{DD} - V_{OUT(\text{opt})} = \frac{1}{2} [V_{DD} - V_{DSAT}] \tag{2.25}$$

Now using the expression for the small-signal gain derived earlier, we have:

$$A_v = -g_m R_D = -\frac{2I_D R_D}{V_{GS} - V_t} \tag{2.26}$$

Evaluating the above expressions at the optimum operating point, we have:

$$A_v = -\frac{V_{DD} - V_{DSAT}}{V_{GS(\text{opt})} - V_t} \tag{2.27}$$

where $V_{GS(\text{opt})}$ is the value of $V_{GS}$ that corresponds to $V_{OUT(\text{opt})}$. Note that this expression has a form similar to that of (2.14) for the BJT case, except
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that the denominator is \( V_{GS(opt)} - V_t \) instead of \( V_T \). This expression, instead of being a physical constant, is a parameter that the designer can control by making an appropriate choice of the transistor \( W/L \), given the chosen bias current. Thus it appears that if this expression could be made sufficiently small, then the gain could be made arbitrarily large. However, for practical reasons to be explained later, this voltage is in practice usually constrained to be no less than about 100 mV.

We will make one more simplifying assumption: \( V_{DSAT} \approx V_{GS(opt)} - V_t \). In other words, we are assuming that \( V_{GS(opt)} \approx V_{GS(\text{eos})} \). For a common-source amplifier with a reasonable gain this assumption is valid since a large change in \( V_{OUT} \) results in a smaller change in \( V_{GS} \), as illustrated in Fig. 2.14(b), where it can be observed that \( V_{GS(opt)} \approx V_{GS(\text{eos})} \). Thus (2.27) can be simplified to:

\[
A_v \approx -\frac{V_{DD} - (V_{GS(opt)} - V_t)}{V_{GS(opt)} - V_t}
\]

\[
= -\left[ \frac{V_{DD}}{V_{GS(opt)} - V_t} - 1 \right]
\]  

(2.28)
**Example 3.2:**

In the common-source amplifier shown in Fig. 2.9(b), let $V_{DD} = 1.8\text{V}$. We wish to bias this amplifier so that $V_{GS} - V_l = 0.3\text{ V}$. Plugging these values into (2.28), we obtain an inverting gain of about 5. (Note that, unlike the BJT case, we need to specify the biasing voltage $V_{GS}$ in order to estimate the gain. Similar to specifying the bias current, a number of constraints, including the amplifier speed and noise generation, will affect the chosen value of this bias voltage.)

To verify this result, let us look at more details of this design:

We begin by choosing $I_D = 250 \mu\text{A}$. (As before, we assume for now that the designer can choose this value arbitrarily.)

In order to achieve the optimum dc setting— that is, setting $V_{OUT} = \frac{3}{2}[V_{DD} + (V_{GS\text{[opt]}} - V_l)] = 1.05\text{ V}$, we should have the following:

$$I_D R_D = V_{DD} - V_{OUT} = 0.75\text{ V}. \quad (2.29)$$

Using the value already chosen for $I_D$, we have:

$$R_D = 0.75\text{ V}/250 \mu\text{A} = 3.0\text{ k}\Omega \quad (2.30)$$

Let us suppose now, based on the transistor model, that the transistor modulation parameter $\lambda = 0.1\text{ V}^{-1}$. Then we have the following small-signal parameters:

$$g_m = \frac{(2)(250 \mu\text{A})}{0.3\text{ V}} = 1.67\text{ m}\Omega$$

$$r_o = \frac{1}{(0.1\text{ V}^{-1})(250 \mu\text{A})} = 40\text{ k}\Omega$$

Applying the above small-signal expressions into (2.17), we can write:

$$A_v = -(1.67\text{ m}\Omega) \cdot (3.0\text{ k}\Omega/40\text{ k}\Omega)$$

$$= -(1.67\text{ m}\Omega) \cdot (2.79\text{ k}\Omega)$$

$$= -4.66$$
2.3. ONE-TRANSISTOR AMPLIFIERS

Note that the above calculations give $R_D|r_o = 2.79$ kΩ, which is within 7% of the value of $R_D$. Thus the assumption that these values are close is still valid, although with less precision than for the common-emitter case. This difference is due to the fact that $1/\lambda$ for this example is an order of magnitude smaller than the $V_A$ of the BJT in previous example.

The design is not complete until the transistor sizing is determined. Let us suppose now, based on the transistor model, that $k' = 150 \mu A/V^2$. Then $W/L$ can be found from the desired dc current $I_D$ and voltage biasing $V_{GS} - V_t$:

\[
\frac{k'}{2} \frac{W}{L} = \frac{250 \mu A}{(0.3 \ V)^2} = 2.78 \times 10^{-3} \ \frac{A}{V^2}
\]

\[
\rightarrow \ \frac{W}{L} = 37
\]

(2.31)

If we now suppose that $V_t = 0.4$ V, then the dc voltage that must be applied to the gate terminal is $(V_{GS} - V_t) + V_t = 0.7$ V.

Notice that our examples suggest that the gain of a typical common-source amplifier is lower by nearly an order of magnitude compared to the gain of a typical common-emitter amplifier. Although the difference between typical values of $V_{CC}$ and $V_{DD}$ plays a role in this behavior, the fundamental reason for this can be seen directly from the graphs in Fig. 2.10. Due to the exponential behavior of a BJT, the common-emitter output voltage decreases much faster than that of the common-source output voltage, which is based on the quadratic behavior of a MOSFET.

Input Resistance

To measure the input resistance of the common-emitter amplifier, we use the schematic shown in Fig. 2.15(a). Consistent with the procedure illustrated in Fig. 2.3 we must now include the load resistor $R_L$ connected between the output and ground. As mentioned earlier, this resistor in general needs to be connected because it is possible for $r_{in}$ to be a function of $R_L$ (although it happens not to be the case here).

We first solve for $i_{in}$, and then find $r_{in}$ using the following equation:

\[
r_{in} = \frac{v_{in}}{i_{in}} \bigg|_{output \ terminated \ with \ R_L}
\]

(2.32)
Figure 2.15: (a) Small-signal schematic for measuring $r_{in}$ of a common-emitter amplifier; (b) common-source amplifier.
2.3. ONE-TRANSISTOR AMPLIFIERS

In Fig. 2.15(a), the only circuit element connected to \( v_{in} \) is resistor \( r_x \) so \( i_{in} \) can be found by inspection: we have \( i_{in} = v_{in}/r_x \), and thus we can simply write \( r_{in} = r_x \). Expressing this quantity in terms of large-signal quantities, we have:

\[
r_{in} = r_x = \frac{\beta V_T}{I_C}
\]  

(2.33)

Input resistance \( r_x \) tends to be a relatively large resistance. The Fig. 2.15(b) common-source amplifier can be analyzed in a similar way. However, notice that in this case \( v_{in} \) is in series with an open circuit, so \( i_{in} \) must be zero. Thus for this amplifier we can write \( r_{in} = \infty \).

Example 3.3:

In the common-emitter amplifier designed in Example 1, the input resistance is given by:

\[
r_{in} = r_x = \frac{\beta V_T}{I_C}
\]  

(2.34)

Using the previously chosen value \( I_C = 500 \mu\text{A} \), \( V_T = 26 \text{ mV} \), and supposing that the transistor model gives \( \beta = 100 \), we have:

\[
r_{in} = \frac{(100)(0.026 \text{ V})}{500 \mu\text{A}} = 5.2 \text{ k\Omega}
\]  

(2.35)

What are the implications of a high \( r_{in} \)? As discussed earlier, a very large input resistance implies that no matter how large the source resistance in series with the source voltage, the voltage that is applied to the input of the amplifier will be almost the same – that is, very little attenuation occurs due to the voltage division that comes from the amplifier’s input. A large input resistance is highly desirable for a voltage amplifier because it doesn’t load the input circuit – that is, very little small-signal current is required.

Output Resistance

To measure the output resistance of the common-emitter amplifier, we use the schematic shown in Fig. 2.16(a). Consistent with the procedure illustrated in Fig. 2.4, for this measurement we terminate the input port with a
load resistor $R_S$ and insert a new voltage source $v_x$ across the output port. We first solve for $i_x$, then find $r_{out}$ using the following equation:

$$r_{out} = \frac{v_x}{i_x \mid_{\text{input terminated with } R_S}}$$  \hspace{1cm} (2.36)

Observing the schematic, we need to use KCL at the collector node to find $i_x$. One of the terms in this KCL equation is the dependent source current $g_m v_\pi$. We can see from the schematic that resistors $R_S$ and $r_\pi$ form a closed loop with no sources connected. Thus voltage $v_\pi$ must be zero. Focusing our attention once again on the output, since the dependent source current $g_m v_\pi$ must be zero, we have simply the parallel combination of $R_C$ and $r_o$ connected across $v_x$. Thus, we can simply write by inspection $r_{out} = R_C || r_o$. As derived earlier in (2.7), we can express this quantity, in terms of large signal parameters:

$$r_{out} = \frac{R_C}{1 + \frac{r_o R_C}{V_A}} \approx R_C$$  \hspace{1cm} (2.37)
2.3. **ONE-TRANSISTOR AMPLIFIERS**

Since $R_C$ is an on-chip resistor, its value will be typically be on the order of kilo-ohms. This is a relatively moderate value of output resistance. As discussed earlier, a small $r_{out}$ is highly desirable in a voltage amplifier.

A similar analysis of the common-source amplifier in Fig. 2.16(b) gives $r_{out} = R_D |r_o$.

---

**Example 3.4:**

In the common-emitter amplifier of Example 3.1, the output resistance is given by:

$$r_{out} = R_C |r_o$$

$$= 3.1 \text{ k}\Omega || 200 \text{ k}\Omega = 3.05 \text{ k}\Omega$$

In the common-source amplifier of Example 3.2, the output resistance is given by:

$$r_{out} = R_D |r_o$$

$$= 3.0 \text{ k}\Omega || 40 \text{ k}\Omega = 2.79 \text{ k}\Omega$$

---

**Summary**

We now summarize the small-signal parameters we have found for the common-emitter and common-source amplifiers:

<table>
<thead>
<tr>
<th>Common-emitter amplifier</th>
<th>Common-source amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_v = -g_m(R_C</td>
<td>r_o)$</td>
</tr>
<tr>
<td>$r_{in} = r_\pi$</td>
<td>$r_{in} = \infty$</td>
</tr>
<tr>
<td>$r_{out} = R_C</td>
<td>r_o$</td>
</tr>
</tbody>
</table>

In general, these configurations exhibit moderate or high inverting gain, large input resistance, and moderate output resistance.
2.3.2 Common-Collector/Common-Drain Amplifier

Corresponding to the sketch in Fig. 2.8(b), where the collector terminal is common to both ports, another one-transistor configuration, the common-collector configuration, is shown in Fig. 2.17(a). Similar to the common-emitter amplifier, the input is again applied to the base terminal, but the output is instead taken at the emitter terminal. In order to bias the BJT in the forward active region, resistor $R_E$ is connected between the emitter and ground in order to allow conduction of emitter current. The common collector terminal is connected to $V_{CC}$ to maintain transistor operation in the forward-active region.

We begin our analysis by examining the qualitative behavior of the common-collector amplifier. Recall from the Fig. 2.9(a) common-emitter amplifier that the input voltage was applied directly across the base-emitter junction, giving rise to an exponential relationship between the input voltage and collector current. However, the topology of the Fig. 2.17(a) common-collector amplifier is different in that voltage $V_B$ is not all dropped across $V_{BE}$; a portion of it is dropped across $R_E$. Using KVL around the loop consisting of $V_B$, the base-emitter junction and $R_E$, we can write:

$$V_B = V_{BE} + I_E R_E$$  \hspace{1cm} (2.38)

We can also express the exponential nature of the transistor’s base-emitter
2.3. ONE-TRANSISTOR AMPLIFIERS

junction as:

\[ I_E = I_{ES} \exp \left( \frac{V_{BE}}{V_T} \right) \]  

(2.39)

We could combine the exact expressions (2.38) and (2.39) but we would find it impossible to solve for the relationship between \( V_B \) and \( I_E \) explicitly. Thus we will need to use some other, approximate, method to understand the circuit’s behavior.

If we solve (2.38) for \( I_E \), we have:

\[ I_E = \frac{1}{R_E} (V_B - V_{BE}) \]  

(2.40)

Consider the two graphs shown in Fig. 2.18(a), which correspond to the two expressions for \( I_E \) written in (2.39) and (2.40). The intersection of the two curves, denoted by label \( X \), determines the dc operating point of the circuit for fixed values of \( V_B \) and \( R_E \). As long as \( V_B > V_{BE(on)} \), this operating point falls well within the region for which the base-emitter junction is turned on, placing it in the high-slope region of the exponential characteristic.

Suppose we now increase the dc input voltage \( V_B \) by a small increment \( \Delta V \). As illustrated in Fig. 2.18(b), this would result in the linear curve corresponding to (2.40) being translated vertically by \( \Delta V/R_E \) as illustrated by the dotted line. As shown in these graphs, this results in a new operating point \( Y \) that is offset from \( X \) vertically by \( \Delta I \) and horizontally by \( \Delta V_{BE} \). It is also evident from the graph that, due to the large slope of the exponential curve, \( \Delta V_{BE} \) is nearly zero; thus \( \Delta I \approx \Delta V/R_E \). Since \( V_{OUT} = I_E R_E \), this increment can be described as follows:

\[ V_{OUT} + \Delta V = R_E (I_E + \Delta I) \]
\[ = I_E R_E + \Delta V \]

Thus \( V_{OUT} \) is a nearly linear function of \( V_B \) - that is, for an increase of \( \Delta V \) at the input, the output increases by nearly \( \Delta V \) as well. From this qualitative analysis we might predict that the small-signal gain will be close to unity. (The reader might wonder why we would be interested in building an amplifier with such a low gain. This will be discussed shortly - at this point we remind you that an amplifier is always designed to provide power gain, which doesn’t always imply a voltage gain.) This analysis also confirms the validity of the assumption that \( V_{BE(on)} \) can be considered constant while the transistor is turned on and an ideal voltage source is not directly connected across the base-emitter junction. Since the emitter voltage “follows” the
Figure 2.18: (a) Graphical analysis of common-collector amplifier biasing; (b) illustration of small-signal behavior.
Figure 2.19: DC voltage transfer characteristic for: (a) common-collector amplifier; (b) common-drain amplifier.

base voltage by a nearly constant amount, this configuration is also known as an emitter follower.

A similar analysis can be done for the Fig. 2.17(b) common-drain amplifier. Because a MOSFET has a quadratic, instead of exponential, characterstic, the plot of $I_D$ vs. $V_{GS}$ exhibits a less sharp characteristic than that of a BJT. Thus, for a small perturbation of the input voltage, $V_{GS}$ tends to change somewhat more than would $V_{BE}$. As a result, the gain of the common-drain amplifier, although near unity, will be somewhat less than that of the common-collector amplifier. It is also consistent with the fact the we generally don’t assume a constant $V_{GS}$ for a MOSFET. This will be analyzed in more detail shortly.

A sketch of the dc voltage transfer characteristics for the common-collector and common-drain amplifiers are shown in Fig. 2.19(a) and (b), respectively. In both cases for the input voltage high enough, a slope of a little less than unity is observed.

**Voltage Gain**

We can analyze the equivalent small-signal diagram shown in Fig. 2.20(a) to find the $A_v$ of the common-collector configuration. Noting that resistors $r_o$ and $R_E$ are connected in parallel, we can write the following KCL equation at the emitter of this circuit:

\[
\frac{v_{out}}{R_E||r_o} - g_m v_n + \frac{1}{r_\pi} (v_{out} - v_{in}) = 0
\]  

(2.41)
Figure 2.20: Small-signal circuit schematic for: (a) common-collector amplifier; (b) common-drain amplifier.
2.3. ONE-TRANSISTOR AMPLIFIERS

In addition, we can express branch voltage \( v_x \) in terms of node voltages:

\[
v_x = v_{in} - v_{out}
\]  

Combining the above two equations gives:

\[
v_{out} \cdot \left[ g_m + \frac{1}{r_x} + \frac{1}{R_E || R_o} \right] = v_{in} \cdot \left[ g_m + \frac{1}{r_x} \right]
\]

\[
\rightarrow A_v \equiv \frac{v_{out}}{v_{in}} = \frac{g_m + \frac{1}{r_x}}{g_m + \frac{1}{r_x} + \frac{1}{R_E || R_o}}
\]

\[
= \frac{1 + \frac{g_m}{r_x}}{1 + \frac{g_m}{r_x} + \frac{1}{R_E || R_o}}
\]  

(2.44)

The last step in the derivation above, dividing top and bottom by \( g_m \) may seem counterintuitive. After all, don’t we want to simplify this expression? The reason for this is that products involving \( g_m \) and a circuit resistance (e.g., \( g_m r_x \)) are often much larger than unity. Thus we can identify the individual expressions in the above expression that are close to zero, making approximation simple and straightforward.

Similar to the analysis done for the analysis of the common-emitter amplifier gain, we first consider the expression for \( R_E || r_o \):

\[
R_E || r_o = \frac{1}{\frac{1}{R_E} + \frac{1}{r_o}}
\]

\[
= \frac{1}{\frac{1}{R_E} + \frac{1}{V_A}}
\]

(2.45)

\[
= \frac{R_E}{1 + \frac{I_E R_E}{V_A}} = \frac{R_E}{1 + \frac{\alpha F I_E R_E}{V_A}}
\]

(2.46)

(2.47)

Once again, the expression in the denominator is the ratio of two voltages: Since emitter current \( I_E \) is conducted by \( R_E \), \( \alpha F I_E R_E \) is slightly less than (since \( \alpha F \) is slightly less than unity) the voltage drop across resistor \( R_E \); \( V_A \) is the transistor Early voltage. For reasons mentioned earlier, it is normally true that this ratio is very small, thus \( R_E || r_o \approx R_E \). We can also recognize from (2.44) the expression \( g_m r_x \), which is identical to the transistor’s small-signal current gain \( \beta \). Thus (2.44) can be rewritten as:

\[
A_v = \frac{1 + \frac{1}{\beta}}{1 + \frac{1}{\beta} + \frac{V_T}{\alpha F I_E R_E} \cdot \left( 1 + \frac{\alpha F I_E R_E}{V_A} \right)}
\]

(2.48)

\[
\approx \frac{1 + \frac{1}{\beta}}{1 + \frac{1}{\beta} + \frac{V_T}{\alpha F I_E R_E}}
\]

(2.49)
From (2.48), we see that in order to ensure that $A_v$ is near unity, we must arrange for $\alpha_F I_E R_E = \alpha_F V_{OUT} >> V_T$. If this condition is satisfied, then the expression in (2.48) is consistent with the qualitative analysis done earlier. Referring to the Fig. 2.17(a) circuit, we can also write:

$$V_{OUT} = V_B - V_{BE(on)} >> V_T$$ (2.50)

In other words, $V_B$ must be selected so that $V_{OUT}$ is biased much higher than $V_T$.

A similar expression for the gain of the common-drain amplifier shown in Fig. 2.17(b) can be derived simply by modifying (2.44) with $r_s = \infty$ and substituting $R_E$ with $R_S$, which gives:

$$A_v = \frac{1}{1 + \frac{1}{g_m(R_S | \frac{r_s}{r_D})}}$$ (2.51)

This expression can be written in terms of biasing parameters by using $g_m = \frac{2I_D}{V_{GS} - V_t}$ and $R_S||r_o = \frac{R_s}{1 + \lambda I_D R_S}$, which gives:

$$A_v = \frac{1}{1 + \frac{V_{GS} - V_t}{2I_D R_S} (1 + \lambda I_D R_S)}$$ (2.52)

As mentioned earlier, it is often the case that $I_D R_S << 1/\lambda$, and thus we can approximate (2.52) as:

$$A_v \approx \frac{1}{1 + \frac{V_{GS} - V_t}{2I_D R_S}} = \frac{1}{1 + \frac{V_{GS} - V_t}{2V_{OUT}}}$$ (2.53)

In order to arrange for $A_v$ to be close to unity, we must bias this amplifier so that $V_{OUT} >> V_{GS} - V_t$. Since we can also write $V_{OUT} = V_G - V_{GS}$, then the inequality can be restated as:

$$V_G >> 2V_{GS} - V_t$$ (2.54)

Thus, for a given value of $V_{GS}$ (determined by the desired bias current $I_D$, transistor dimensions, $V_t$ and $k^l$), dc input voltage $V_G$ must be chosen to satisfy (2.54).
2.3. ONE-TRANSISTOR AMPLIFIERS

Example 3.5:
We wish to design a common-collector amplifier with $V_{CC} = 3.3$ V whose dc output voltage is 1.65 V and dc collector current is 1 mA. Assume that the transistor $\beta = 75$.

Referring to the common-collector amplifier schematic shown in Fig. 2.9(a), we can write:

$$I_E = \frac{I_C}{\alpha_F} = \frac{V_{OUT}}{R_E}$$
$$\rightarrow R_E = \frac{\alpha_F V_{OUT}}{I_C} = \frac{(0.987)(1.65 \text{ V})}{1 \text{ mA}} = 1.63 \text{ k}\Omega$$

To find the appropriate value for the input voltage we note that $V_{IN} = V_{OUT} + V_{BE(on)} \approx 1.65 + 0.7 = 2.35$ V. Note that this value of $V_{IN}$ satisfies (2.50).

We now calculate the small-signal gain for this circuit. Using (2.48), and supposing that the transistor model gives $V_A = 80$, we have:

$$A_v = \frac{1 + \frac{1}{\beta}}{1 + \frac{1}{\beta} + \frac{0.026 \text{ V}}{(0.987)(1.65 \text{ V})} \times \left(1 + \frac{(0.987)(1.65 \text{ V})}{80 \text{ V}}\right)}$$
$$A_v = \frac{1 + 0.013}{1 + 0.013 + (0.016)(1.02)} = 0.984$$

The factor 1.02 in the denominator of the above expression confirms that for this example the approximation in (2.49) holds.
Example 3.6:

We wish to design a common-drain amplifier with $V_{DD} = 1.8 \, \text{V}$ whose dc output voltage is 0.9 V, dc drain current is 300 $\mu\text{A}$, and $V_{GS} - V_t = 0.4 \, \text{V}$. Assume the transistor model gives $V_t = 0.4 \, \text{V}$ and $k' = 150 \, \mu\text{A}/\text{V}^2$.

Referring to the common-drain amplifier schematic shown in Fig. 2.9(b), we can write:

$$I_D = \frac{V_{OUT}}{R_S}$$

$$\rightarrow R_S = \frac{V_{OUT}}{I_D} = 3 \, \text{k}\Omega$$

We now calculate the small-signal gain for this circuit. Using (2.52) and supposing that the transistor model gives $\lambda = 0.15 \, \text{V}^{-1}$, we have:

$$A_v = \frac{1}{1 + \frac{0.4 \, \text{V}}{(2)(0.9 \, \text{V})} \cdot (1 + (0.15 \, \text{V}^{-1})(0.9 \, \text{V}))}$$

$$= \frac{1}{1 + (0.11)(1 + 0.135)}$$

$$= 0.799$$

Note that this value is about 20% lower than that for the common-collector amplifier, mainly because of relative large value of $V_{GS} - V_t$ that was chosen.

To determine the appropriate transistor dimensions $W/L$, we use the following:

$$300 \, \mu\text{A} = \frac{150 \, \mu\text{A}/\text{V}^2}{2} \left(\frac{W}{L}\right) \cdot (0.4 \, \text{V})^2$$

$$\rightarrow \frac{W}{L} = 25$$

To determine the required input voltage, we use the following:

$$V_{IN} = V_{OUT} + V_{GS}$$

$$= 0.9 \, \text{V} + 0.4 \, \text{V} + 0.4 \, \text{V} = 1.7 \, \text{V}$$
2.3. ONE-TRANSISTOR AMPLIFIERS

Input Resistance
To determine the input resistance of the common-collector amplifier, we use the schematic shown in Fig. 2.21(a). As before, load resistor $R_L$ has been placed in parallel with the output as required by the definition of $r_{in}$. Notice also that $r_o$ is in parallel with $R_L$ and $R_E$.

To analyze $r_{in}$ for this circuit we first write a KCL equation at the input:

$$i_{in} = \frac{1}{r_{\pi}} (v_{in} - v_{out})$$

$$\rightarrow \frac{i_{in}}{v_{in}} = \frac{1 - \frac{v_{out}}{v_{in}}}{r_{\pi}}$$  \hspace{1cm} (2.55)

Before going any further, recall that we have already found an expression for $v_{out}/v_{in}$ in (2.44) for the case where there was no load resistance connected to the circuit. However, the Fig. 2.21(a) and Fig. 2.17(a) schematics differ only in that $R_L$ is connected in parallel to $R_E$. Hence, we can simply modify (2.44) to apply to the Fig. 2.21(a) circuit by replacing $(R_E||r_o)$ with $(R_E||r_o||R_L)$:

$$\frac{v_{out}}{v_{in}} = 1 + \frac{1}{g_m r_{\pi}} + \frac{1}{g_m (R_E||r_o||R_L)}$$

$$\rightarrow 1 - \frac{v_{out}}{v_{in}} = 1 + \frac{1}{g_m (R_E||r_o||R_L)}$$ \hspace{1cm} (2.56)

Substituting (2.56) back into (2.55), we can write:

$$i_{in} = \frac{1}{r_{\pi}} \cdot \frac{1}{1 + \frac{1}{g_m (R_E||r_o||R_L)}}$$

$$\rightarrow \frac{r_{in}}{i_{in}} = r_{\pi} \cdot \frac{1 + \frac{1}{g_m (R_E||r_o||R_L)}}{1 + \frac{1}{g_m (R_E||r_o||R_L)}}$$

$$= g_m r_{\pi} (R_E||r_o||R_L) + R_E||r_o||R_L + r_{\pi}$$  \hspace{1cm} (2.57)

Recognizing the identity $g_m r_{\pi} = \beta$, we can rewrite this expression as:

$$r_{in} = (\beta + 1)(R_E||r_o||R_L) + r_{\pi}$$  \hspace{1cm} (2.58)

We can interpret this expression as two resistances in series: $r_{\pi}$, which is seen directly in series with the input, and the parallel combination of $R_E$, $r_o$, and $R_L$, all of which are connected between the emitter and ground, multiplied by $\beta + 1$. In other words, any resistance connected between the
Figure 2.21: (a) Schematic for measuring $r_{in}$ of a common-collector amplifier; (b) common-drain amplifier.
emitter and ground appears, looking from the base, as if it is multiplied by
a \( \beta + 1 \), normally a large number. For this reason, the \( r_{in} \) of an emitter
follower amplifier is generally a large value. Note that for this circuit the
value of \( r_{in} \) does depend on load resistance \( R_L \).

To see how \( r_{in} \) is affected by biasing parameters, we can rewrite the
parallel resistance expression as:

\[
R_E || r_{in} || R_L = \frac{R_E || R_L}{1 + \frac{I_C(R_E || R_L)}{V_A}}
\]

Thus from (2.58) the expression for \( r_{in} \) in terms of biasing parameters can
be written as:

\[
r_{in} = (\beta + 1) \frac{R_E || R_L}{1 + \frac{I_C(R_E || R_L)}{V_A}} + \frac{\beta V_T}{I_C}
\]

Making the usual assumptions that \( \beta >> 1 \) and \( I_C(R_E || R_L) << V_A \) (since
this is the dc drop across \( R_L \)), we can simplify the above expression to:

\[
r_{in} \approx \beta \left[ R_E || R_L + \frac{V_T}{I_C} \right]
\]

Because the above expression is proportional to the transistor \( \beta \), the input
resistance is in general relatively large.

For the common-drain amplifier shown in Fig. 2.21(b), the gate appears
as an open circuit; thus, without any further calculation, we can write:

\[
r_{in} = \infty
\]

**Example 3.7:**

In the common-collector amplifier designed in Example 3.5, we will calculate
its input resistance under the condition that the output is terminated with
a load resistance \( R_L = 5 \text{ k}\Omega \). Using the expression in (2.60) and supposing
that the transistor model gives \( \beta = 75 \) and \( V_A = 80 \text{ V} \), we have:

\[
r_{in} = (76) \frac{1.63 \text{ k}\Omega || 5 \text{ k}\Omega}{1 + \left( \frac{1 \text{ mA}(1.63 \text{ k}\Omega || 5 \text{ k}\Omega)}{80 \text{ V}} \right)} + \frac{(75)(0.026 \text{ mV})}{1 \text{ mA}}
\]

\[
= 92 \text{ k}\Omega + 1.95 \text{ k}\Omega
\]

\[
= 94.9 \text{ k}\Omega
\]
Output Resistance
To measure the output resistance of the common-collector amplifier, we use the schematic shown in Fig. 2.22(a). For this measurement we replace the input voltage source with a load resistor \( R_{\text{source}} \) and instead apply a new voltage source \( v_x \) across the output.

We begin this analysis by writing the following KCL equation at the emitter node:

\[
i_x = \frac{v_x}{R_E|r_o} - g_m v_x + \frac{1}{r_\pi} (v_x - v_b)
\]  
(2.63)

We can also express \( v_\pi \) in terms of node voltages:

\[
v_\pi = v_b - v_x
\]  
(2.64)

Combining the above two equations, we have:

\[
i_x = \frac{v_x}{R_E|r_o} + \left( g_m + \frac{1}{r_\pi} \right) (v_x - v_b)
\]  
(2.65)

In order to express \( v_b \) in terms of \( v_x \), we can observe that the series connection of \( r_\pi \) and \( R_{\text{source}} \) creates a voltage divider. Thus, we can simply write:

\[
v_b = \frac{R_{\text{source}}}{R_{\text{source}} + r_\pi} \cdot v_x
\]  
(2.66)

\[
\rightarrow v_x - v_b = \frac{r_\pi}{R_{\text{source}} + r_\pi} \cdot v_x
\]  
(2.67)

Substituting (2.67) into (2.65), we have:

\[
i_x = \frac{v_x}{R_E|r_o} \left[ \frac{1}{R_{\text{source}}|r_o} + \frac{1 + g_m r_\pi}{R_{\text{source}} + r_\pi} \right]
\]  
(2.68)

\[
\rightarrow r_{out} = \frac{v_x}{i_x}
\]  
(2.69)

\[
= R_{E|r_o} \left[ \frac{R_{\text{source}} + r_\pi}{1 + g_m r_\pi} \right] \left[ \frac{1}{\beta + 1} \right]
\]  
(2.70)

We can interpret this expression as two resistances in parallel: \( (R_E|r_o) \), which is connected directly between the emitter and ground, and the series combination of \( R_{\text{source}} \) and \( r_\pi \), which is connected in series with the base, divided by \( \beta + 1 \). In other words, any resistance connected in series with the base appears from the emitter as if it is divided by \( \beta + 1 \). For this reason the \( r_{out} \) of an emitter follower is generally a small value. Notice that for this amplifier \( r_{out} \) does depend on the value of source resistance \( R_{\text{source}} \).
Figure 2.22: (a) Schematic for measuring $r_{out}$ of a common-collector amplifier; (b) common-drain amplifier.
Writing the above expression in terms of biasing parameters and using $g_m r_x = \beta$, we have:

$$r_{out} = \left[ \frac{R_E}{1 + \frac{I_C R_E}{V_A}} \right] \left[ \frac{R_{source} + \frac{\beta V_T}{I_C}}{1 + \beta} \right]$$

(2.71)

Assuming $\beta >> 1$ and $I_C R_E << V_A$, we can express $r_{out}$ as:

$$r_{out} \approx R_E \left[ \frac{V_T}{I_C} + \frac{R_{source}}{\beta} \right]$$

(2.72)

This resistance is relatively small.

A similar analysis can be performed to find the output resistance of the common-drain amplifier as shown in Fig. 2.22(b). Notice that the topology is identical to that of Fig. 2.22(a), with $R_E$ replaced by $R_S$ and $\beta$ going to infinity. To determine this, let us examine the last term in (2.69) where we let $r_x = \beta/g_m$ and $\beta \to \infty$:

$$r_{out} = (R_S \parallel r_o) \parallel \frac{1}{g_m}$$

(2.73)

In the usual case where $g_m r_o$ and $g_m R_S$ are both much larger than 1, $r_{out} \approx 1/g_m$.

Substituting biasing parameters in the above equations, we have:

$$r_{out} = \left[ \frac{R_S}{1 + \lambda I_D R_S} \right] \left[ \frac{V_{GS} - V_t}{2I_D} \right]$$

(2.74)

Assuming $\lambda I_D R_S << 1$, this can be approximated as:

$$r_{out} \approx R_S \left[ \frac{V_{GS} - V_t}{2I_D} \right]$$

(2.75)
2.3. **ONE-TRANSISTOR AMPLIFIERS**

**Example 3.8:**
In the common-collector amplifier designed in Example 3.5, we will calculate its output resistance under the condition that the input is terminated with a source resistance $R_{source} = 2 \, \text{k}\Omega$.

Using the expression in (2.71) and supposing the BJT transistor model has $\beta = 75$ and $V_A = 80$, we have:

\[
\begin{align*}
    r_{out} &= \frac{1.63 \, \text{k}\Omega}{1 + \frac{0.987(1.65 \, \text{V})}{80 \, \text{V}}} \ \parallel \ \left[ \frac{2 \, \text{k}\Omega + \frac{(75)(0.026 \, \text{V})}{1 \, \text{mA}}}{76} \right] \\
    &= 620 \, \Omega \ \parallel 52 \, \Omega = 48 \, \Omega
\end{align*}
\]

**Example 3.9:**
In the common-drain amplifier designed in Example 3.6, we will calculate its output resistance under the condition that the input is terminated with a source resistance $R_{source} = 2 \, \text{k}\Omega$. Using the expression in (2.74) and supposing the MOS transistor model has $\lambda = 0.15 \, \text{V}^{-1}$, we have:

\[
\begin{align*}
    r_{out} &= \frac{3 \, \text{k}\Omega}{1 + (0.15 \, \text{V}^{-1})(0.9 \, \text{V})} \ \parallel \ \left[ \frac{0.4 \, \text{V}}{(2)(300 \, \mu\text{A})} \right] \\
    &= 2.64 \, \text{k}\Omega \ \parallel 667 \, \Omega = 532 \, \Omega
\end{align*}
\]

**Summary**
Let us summarize the small-signal parameters we have found for the common-collector and common-drain amplifiers:
CHAPTER 2. AMPLIFICATION

<table>
<thead>
<tr>
<th>Common-collector amplifier</th>
<th>Common-drain amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ A_v = \frac{1 + \beta}{1 + \frac{\beta}{g_m R_E}</td>
<td></td>
</tr>
<tr>
<td>[ r_{in} = (\beta + 1) (R_E</td>
<td>R_L</td>
</tr>
<tr>
<td>[ r_{out} = R_E</td>
<td></td>
</tr>
</tbody>
</table>

Notice that the input resistance is typically quite large. The output resistance contains a parallel term that is inversely proportional to \( g_m \); thus it is typically quite small. This brings us to the purpose of this amplifier configuration: its input impedance is quite large and its output impedance is quite small. Even though the voltage gain is close to unity, its power gain, as discussed in the previous chapter, can be quite large. For this reason this configuration is normally used as a voltage buffer.

2.3.3 Common-Base/Common-Gate Amplifier

Corresponding to the sketch in Fig. 2.8(c), the common-base configuration is shown in Fig. 2.23(a). In this case there are three dc biasing sources: voltage sources at the collector and base, used to set the transistor in the forward active region, and a current source conducted from the emitter to ground. The small-signal input \( i_{in} \) is also a current source, and is placed in parallel with the dc current source. The output signal \( i_{out} \) is a current as well, conducted into the collector terminal. The common-base terminal and collector terminal are connected to dc voltage sources to maintain biasing of the transistor in the forward-active region.

Conceptually the operation of the Fig. 2.23(a) common-base amplifier is very simple. Assuming the transistor is biased in the forward-active region, any change \( i_{in} \) of the emitter current will result in a change of approximately \( a i_{in} \) in the collector current. (We say approximately here because changing the collector-emitter current will cause a slight change in the emitter voltage, which itself will also affect the collector current. This will be analyzed in more detail shortly.) Thus we expect the current gain \( A_i \) to be approximately \( a \). The Fig. 2.23(b) common-gate amplifier operates in a similar way, but with the source and drain currents identical; thus for this configuration, \( A_i = 1 \) exactly.

Current Gain

Using the Fig. 2.24(a) small-signal schematic and recognizing that \( r_o \) and \( r_\pi \) are connected in parallel, we can write the following KCL equation at the
2.3. **ONE-TRANSISTOR AMPLIFIERS**

![Diagram of amplifiers](image)

Figure 2.23: (a) Common-base amplifier; (b) common-gate amplifier.

 emitter:

\[ i_{in} + \frac{v_e}{r_{\pi}r_o} - g_m v_{\pi} = 0 \] \hspace{1cm} (2.76)

We can also write an expression for branch voltage \( v_{\pi} \) in terms of node voltages:

\[ v_{\pi} = -v_e \] \hspace{1cm} (2.77)

Combining the above two equations, we have:

\[ v_e = -\frac{i_{in}}{g_m + \frac{1}{r_{\pi}r_o}} = -\frac{i_{in}}{g_m + \frac{1}{r_{\pi}} + \frac{1}{r_o}} \] \hspace{1cm} (2.78)

Now that we’ve solved for node voltage \( v_e \), we will express output current \( i_{out} \) by writing a KCL equation at the collector node:

\[ i_{out} = g_m v_{\pi} - \frac{v_e}{r_o} = -v_e \left( g_m + \frac{1}{r_o} \right) \] \hspace{1cm} (2.79)

Combining (2.78) and (2.79), we have:

\[ i_{out} = i_{in} \cdot \frac{g_m + \frac{1}{r_o}}{g_m + \frac{1}{r_{\pi}r_o} + \frac{1}{r_{\pi}}} = \frac{i_{in} \cdot \left( 1 + \frac{1}{g_m r_o} \right)}{1 + \frac{1}{g_m r_o} + \frac{1}{g_m r_{\pi}}} \] \hspace{1cm} (2.80)
Figure 2.24: Small-signal schematic of: (a) common-base amplifier; (b) common-gate amplifier.
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Recognizing the identity $g_{m}r_{p} = \beta$ and replacing $g_{m}r_{o}$ with $V_{A}/V_{T}$ in the above expression, we can write:

$$A_{i} = \frac{1 + \frac{V_{T}}{V_{A}}}{1 + \frac{1}{\beta} + \frac{V_{T}}{V_{A}}} \quad (2.81)$$

This expression includes two quantities, $V_{T}/V_{A}$ and $1/\beta$, that are usually much less than unity. Using typical values of $\beta = 100$, $V_{T} = 26\text{mV}$, and $V_{A} = 100\text{V}$, we have:

$$\frac{V_{A}}{V_{T}} \approx 3800 \gg \beta \quad (2.82)$$

and thus we can write:

$$\frac{V_{T}}{V_{A}} \ll \frac{1}{\beta} \ll 1 \quad (2.83)$$

As a result, (2.81) can be approximated as:

$$A_{i} \approx \frac{1}{1 + \frac{1}{\beta}} = \alpha_{F} \quad (2.84)$$

This confirms our previous qualitative analysis where we predicted that the current gain would be close to unity.

For the common-gate small-signal schematic shown in Fig. 2.24(b), we can see by inspection that KCL dictates $i_{out} = i_{in}$; i.e., $A_{i} = 1$ exactly. Note that, unlike the BJT case, this expression is independent of $r_{o}$.

The common-base or common-gate amplifier is usually used as a unity-gain current buffer, and its operation is analogous to the operation of the common-collector/common-drain amplifier for voltage signals. As with that configuration, the usefulness of this amplifier will become apparent after analyzing the input and output resistances.
Example 3.10:
In the common-base amplifier shown in Fig. 2.23(a), using $V_{CC} = 3.3$ V, design the circuit for $I_E = 500$ μA with the input voltage (at the emitter) biased at 1.0 V. Suppose that the transistor model gives $\beta = 100$ and $V_A = 80$.

The only design parameter required here is the base bias voltage $V_B$, which will simply be one $V_{BE(on)}$ higher than the desired dc voltage at the emitter. Thus we set $V_B = 1.7$ V.

To find the small-signal current gain we use the expression in (2.84):

\[
A_i = \frac{1 + \frac{0.026 \text{ V}}{80 \text{ V}}}{1 + \frac{0.026 \text{ V}}{80 \text{ V}}} = \frac{1 + 0.0003}{1 + 0.01 + 0.0003} = 0.99
\]
2.3. ONE-TRANSISTOR AMPLIFIERS

Example 3.11:
In the common-gate amplifier shown in Fig. 2.23(b), using \( V_{DD} = 1.8 \) V, design the circuit for \( I_S = 250 \ \mu A \) and \( V_{GS} - V_t = 0.8 \) V with the input voltage (at the source) biased at 0.4 V. Suppose from the transistor model that \( k' = 150 \ \mu A/V^2 \) and \( V_t = 0.4 \) V.

The two design parameters that need to be found are the transistor \( W/L \) and the gate bias voltage \( V_G \). As shown in previous examples, the \( W/L \) can be found directly from the desired bias current and \( V_{GS} - V_t \):

\[
\frac{2I_D}{k'(V_{GS} - V_t)^2} = \frac{(2)(250 \ \mu A)}{(150 \ \mu A/V^2)(0.8 V^2)^2}
\]

\[
W/L = 5.2
\]

To find the value of \( V_G \), we have:

\[
V_G = 0.4 \ \text{V} + V_{GS}
\]

\[
= 0.4 \ \text{V} + 0.8 \ \text{V} + 0.4 \ \text{V}
\]

\[
= 1.6 \ \text{V}
\]

As mentioned earlier, the current gain of the common-gate amplifier is always exactly unity.

Input Resistance
The small-signal schematic used for calculating the input resistance of the common-base configuration is shown in Fig. 2.25(a). Notice that since the output signal is a current, the load resistance \( R_L \) is connected in series with the output branch. Before writing equations, we can first recognize that resistor \( r_m \) is connected in parallel with current source \( i_x \). Thus the input resistance of this circuit must consist of \( r_m \) in parallel with the equivalent resistance \( r_m' \) posed by the other elements in the circuit. Referring to Fig. 2.25(a), we’ll begin with the more simple analysis of finding \( r_m' \equiv v_e/i_x' \) where \( i_x' \) is defined as shown in Fig. 2.25(a). Writing the KCL equation at the emitter, we have:

\[
-i_x' - g_m v_x + \frac{1}{r_p} (v_e - v_x) = 0
\]
Figure 2.25: $r_{in}$ calculation for: (a) common-base amplifier; (b) common-gate amplifier.
2.3. ONE-TRANSISTOR AMPLIFIERS

We can also write:

\[ v_x = -v_e \]  \hspace{1cm} (2.86)

In addition, KCL requires that the current conducted in \( R_L \) must also be \( i'_x \); thus we can write:

\[ v_e = i'_x R_L \]  \hspace{1cm} (2.87)

Substituting (2.87) and (2.86) into (2.85), we have:

\[ -i_x + g_m v_e + \frac{1}{r_o} (v_e - i_x R_L) = 0 \]

\[ \rightarrow v_e \left( g_m + \frac{1}{r_o} \right) = i_x \left( 1 + \frac{R_L}{r_o} \right) \]

\[ r'_\text{in} \equiv \frac{v_e}{i_x} = \frac{1 + \frac{R_L}{r_o}}{g_m + \frac{1}{r_o}} \]

\[ = \frac{1}{g_m} \cdot \frac{1 + \frac{R_L}{r_o}}{1 + \frac{1}{g_m r_o}} \]  \hspace{1cm} (2.88)

To express \( r'_\text{in} \) in terms of biasing parameters, we can write the following:

\[ \frac{R_L}{r_o} = \frac{R_L}{V_A} \cdot \frac{I_C}{V_A} = \frac{I_C R_L}{V_A} \]  \hspace{1cm} (2.89)

\[ \frac{1}{g_m r_o} = \frac{V_T}{I_C} \cdot \frac{I_C}{V_A} \cdot \frac{V_T}{V_A} \]  \hspace{1cm} (2.90)

Substituting the above expression into (2.88), we have:

\[ r'_\text{in} = \frac{1}{g_m} \cdot \frac{1 + \frac{I_C R_L}{V_A}}{1 + \frac{V_T}{V_A}} \]  \hspace{1cm} (2.91)

\[ \rightarrow r_{\text{in}} = r_x \left| \frac{1}{g_m} \cdot \frac{1 + \frac{I_C R_L}{V_A}}{1 + \frac{V_T}{V_A}} \right| \]  \hspace{1cm} (2.92)

Note that since \( R_L \) is in series with the collector, \( I_C R_L \) is the dc voltage drop across \( R_L \). Thus we can assume \( I_C R_L << V_A \) and \( V_T << V_A \), which gives:

\[ r_{\text{in}} \approx \frac{r_x}{g_m} \left| \frac{1}{g_m} \right| \]  \hspace{1cm} (2.93)

\[ = \frac{\beta V_T}{I_C} \left| \frac{V_T}{I_C} \right| \]  \hspace{1cm} (2.94)

\[ = \alpha \frac{V_T}{I_C} \]  \hspace{1cm} (2.95)
We see that the input resistance of a common-base amplifier is very close to \(1/g_m\). The resistance \(r_e||\frac{1}{g_m}\) occurs often in expressions for BJT amplifiers and sometimes referred to as \(r_e\), which is the small-signal resistance seen at the emitter of a BJT with the base grounded. For hand calculations this is often approximated by \(1/g_m\) under the assumption that \(\beta >> 1\).

We can directly find the expression for the input resistance of the common-gate amplifier (corresponding to the small-signal schematic in Fig. 2.25(b)) by observing (2.88) and recognizing that \(r_e\) becomes an open circuit in a MOSFET. Thus we can directly write:

\[
\begin{align*}
    r_{in} &= \frac{1}{g_m} \cdot \frac{1 + \frac{R_L}{r_o}}{1 + \frac{1}{g_m r_o}} \\
    &= \frac{1}{g_m} \cdot \frac{1 + \frac{R_L}{r_o}}{1 + \frac{1}{g_m r_o}}.
\end{align*}
\]

(2.96)

We can also write:

\[
\begin{align*}
    \frac{R_L}{r_o} &= \lambda I_D R_L, \\
    \frac{1}{g_m r_o} &= \frac{\lambda (V_{GS} - V_t)}{2}.
\end{align*}
\]

(2.97)

Substituting the above expressions into (2.96), we have:

\[
\begin{align*}
    r_{in} &= \frac{1}{g_m} \cdot \frac{1 + \frac{\lambda I_D R_L}{1 + \frac{\lambda (V_{GS} - V_t)}{2}}}{1 + \frac{\lambda (V_{GS} - V_t)}{2}} \\
    &= \frac{1 + \frac{\lambda I_D R_L}{1 + \frac{\lambda (V_{GS} - V_t)}{2}}}{1 + \frac{\lambda (V_{GS} - V_t)}{2}}.
\end{align*}
\]

(2.98)

Assuming \(\lambda I_D R_L\) and \(\lambda (V_{GS} - V_t)\) are both very small, we can write:

\[
r_{in} \approx \frac{1}{g_m}
\]

(2.99)
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Example 3.12:
In the common-base amplifier designed in Example 3.10, we wish to find
the input resistance when the output is terminated by 500 Ω. Using the
expression for \( r_{in} \) in (2.92), we first calculate the small-signal parameters \( r_\pi \)
and \( g_m \):

\[
\begin{align*}
  r_\pi &= \frac{\beta V_T}{I_C} = \frac{\beta V_T}{\alpha_F I_E} \nonumber \\
  &= \frac{(100)(0.026 \text{ V})}{(0.99)(500 \mu\text{A})} \\
  &= 5.25 \text{kΩ} \\

  g_m &= \frac{I_C}{V_T} = \frac{\alpha_F I_E}{V_T} \\
  &= \frac{(0.99)(500 \mu\text{A})}{0.026 \text{ V}} \\
  &= 19 \text{ mS} \\

  \Rightarrow \quad \frac{1}{g_m} &= 52.5 \text{ Ω}
\end{align*}
\]

We then substitute these values into the (2.92) expression:

\[
\begin{align*}
r_{in} &= \left[ 5.25 \text{kΩ} \right] \left[ 52.5 \text{ Ω} \cdot \frac{1 + \frac{(0.99)(500 \mu\text{A})(500 \text{ Ω})}{80 \text{ V}}}{1 + \frac{0.026 \text{ V}}{80 \text{ V}}} \right] \\
  &= 5.25 \text{kΩ} \left[ 52.5 \text{ Ω} \cdot \frac{1 + 0.003}{1 + 0.0003} \right] \\
  &= 52.6 \text{ Ω}
\end{align*}
\]

Note that this resistance is very close to \( 1/g_m \), as predicted by the approximation in the text.
Example 3.13:
In the common-gate amplifier designed in Example 3.11, we wish to find the input resistance when the output is terminated by 750 Ω. Using the expression for $r_{in}$ in (2.98), we first calculate the transistor $g_m$:

$$g_m = \frac{2I_D}{V_{GS} - V_t} = \frac{(2)(250 \mu A)}{0.8 V} = 625 \mu S$$

$$\rightarrow \frac{1}{g_m} = 1.6 \text{ k}\Omega$$

Substituting this value into (2.98) and supposing that the transistor model gives $\lambda = 0.1 \text{ V}^{-1}$, we have:

$$r_{in} = 1.6 \text{ k}\Omega \cdot \frac{1 + (0.1 \text{ V}^{-1})(250 \mu A)(750 \text{ Ω})}{1 + (0.1 \text{ V}^{-1})(0.8 \text{ rmV})} = 1.6 \text{ k}\Omega \cdot \frac{1 + 0.019}{1 + 0.04} = 1.57 \text{ k}\Omega$$

Note that this resistance is also very close to $1/g_m$.

Output Resistance
The small-signal schematic used to calculate the output resistance of a common-base amplifier is shown in Fig. 2.26(a). As usual, source resistance $R_S$ has been inserted across the input port. To begin our analysis of this circuit we can write, after replacing $v_x$ with $-v_e$, the following KCL equation at the collector:

$$i_x = \frac{1}{r_o} (v_x - v_e) - g_m v_e$$  \hspace{1cm} (2.100)$$

As usual, since there are two unknowns in this circuit, we need a second equation, which would naturally be the KCL equation at the emitter. However, in this case we can take a shortcut. As illustrated in Fig. 2.26(a), KCL dictates that the current coming out of the parallel combination of the dependent current source and $r_o$ must be $i_x$. Thus, we can write by inspection:

$$v_e = i_x \cdot (r_\pi || R_S)$$  \hspace{1cm} (2.101)$$
2.3. **ONE-TRANSISTOR AMPLIFIERS**

Figure 2.26: $r_{out}$ calculation: (a) common-base amplifier; (b) common-gate amplifier.
Combining (2.100) and (2.101), we have:

\[ i_x = \frac{v_x}{v_0} - \left( \frac{1}{v_0} + g_m \right) \left[ i_x \cdot (r_x | R_S) \right] \]  

(2.102)

Solving the above equation, we have:

\[ r_{out} = \frac{v_x}{i_x} = R_S | r_x + r_o + g_m r_o (R_S | r_x) \]  

(2.103)

Based on our previous analyses, at this point it would be tempting to, as usual, find an approximation that gives us some insight into the expression \( R_S | r_x \):

\[ R_S | r_x = \frac{1}{\frac{1}{R_S} + \frac{I_C}{\beta V_T}} \]

\[ = \frac{R_S}{1 + \frac{I_C R_S}{\beta V_T}} \]  

(2.104)

As usual, in the denominator we see the ratio of two voltages: \( I_C R_S \) and \( V_T \). However, the \( I_C R_S \) term is not a dc voltage drop found in the circuit. The reason for this is that the dc current conducted out of the emitter is mostly from the input current source; very little dc current would be typically conducted in the parallel source resistance \( R_S \). Thus we cannot make any general statements about this voltage ratio unless we know more about the value of \( R_S \). Instead, we will consider two cases corresponding to different ranges of \( R_S \):

**Case 1: \( r_x << R_S \)**

Since, for this case \( R_S | r_x \approx r_x \), we can write from (2.103):

\[ r_{out} \approx r_x + r_o + g_m r_x r_o \]

\[ = r_x + r_o + \beta r_o \]  

(2.105)

Assuming \( \beta >> 1 \) and \( \beta r_o >> r_x \), we can write:

\[ r_{out} \approx \beta r_o \]  

(2.106)

**Case 2: \( r_x >> R_S \)**

Since, for this case, \( R_S | r_x \approx R_S \), we can write from (2.103):

\[ r_{out} \approx R_S + r_o + g_m R_S r_o \]

\[ = r_o + R_S (1 + g_m r_o) \]  

(2.107)
2.3. ONE-TRANSISTOR AMPLIFIERS

Since generally \( g_m r_o \gg 1 \), then \( g_m R_s r_o \gg R_s \) and thus we can write:

\[
\begin{align*}
  r_{out} & \approx r_o + g_m R_s r_o \\
           & = r_o (1 + g_m R_s) \\
           & = r_o \left[ 1 + \frac{I_C R_s}{V_T} \right] \\
\end{align*}
\]

(2.108)

As mentioned before, we can’t say in general how \( I_C R_s \) compares to \( V_T \). However, it is certainly true that \( r_{out} > r_o \).

For determining the \( r_{out} \) of the common-gate amplifier corresponding to Fig. 2.26(b), we replace \( r_x \) with an open circuit, which corresponds to (2.108) above. Thus we can write:

\[
\begin{align*}
  r_{out} & = r_o (1 + g_m R_s) \\
           & = r_o \left[ 1 + \frac{2I_D R_s}{V_{GS} - V_t} \right] \\
\end{align*}
\]

(2.109)

Similar to the common-collector/common-drain configurations, the common-base/common-gate amplifier has near unity current gain, but the presence of greatly different input and output resistances allows for power gain.
Example 3.14:

In the common-base amplifier designed in Example 3.10, we wish to find the output resistance when the input is terminated by 100 Ω. Using the expression for \( r_{out} \) in (2.103), we use the small-signal parameters calculated in Example 12: \( g_m = 19 \text{ mS}, r_\pi = 52.5 \text{ kΩ} \). We also calculate the transistor \( r_o \):

\[
r_o = \frac{V_A}{I_C}
\]

\[
= \frac{80 \text{ V}}{(0.99)(500 \text{ µA})}
\]

\[
= 162 \text{ kΩ}
\]

We then substitute these values into the (2.103) expression:

\[
r_{out} = [100 \text{ Ω} \parallel 52.5 \text{ kΩ}] + 162 \text{ kΩ} \\
+ (19 \text{ mS})(162 \text{ kΩ}) [100 \text{ Ω} \parallel 52.5 \text{ kΩ}] \\
= 100 \text{ Ω} + 162 \text{ kΩ} + 307 \text{ kΩ} \\
= 469 \text{ kΩ}
\]
2.3. ONE-TRANSISTOR AMPLIFIERS

Example 3.15:
In the common-gate amplifier designed in Example 3.13, we wish to find the output resistance when the input is terminated by 200 \( \Omega \). Using the expression for \( r_{out} \) in (2.109), we first calculate the transistor \( r_o \):

\[
\frac{1}{r_o} = \frac{1}{\lambda I_D} = \frac{1}{(0.1 \text{ V}^{-1})(250 \text{ \mu A})} = 40 \text{ k}\Omega
\]

We then substitute these values into the (2.109) expression:

\[
\frac{1}{r_{out}} = \left(40 \text{ k}\Omega\right) \left[1 + \frac{(2)(250 \text{ \mu A})(200 \text{ \Omega})}{0.8 \text{ V}}\right] = 45 \text{ k}\Omega
\]

Summary

Let us summarize the small-signal parameters we have found for the common-base and common-gate amplifiers:

<table>
<thead>
<tr>
<th>Common-base</th>
<th>Common-gate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_i = \frac{1 + \frac{1}{g_m} + \frac{1}{g_m r_o}}{1 + \frac{1}{g_m} + \frac{1}{g_m r_o}} \approx 1 )</td>
<td>( A_i = 1 )</td>
</tr>
<tr>
<td>( r_{in} = \frac{1}{g_m} \cdot \frac{1}{1 + \frac{1}{g_m} + \frac{1}{g_m r_o}} \approx \frac{1}{g_m} )</td>
<td>( r_{in} = \frac{1}{g_m} \cdot \frac{1}{1 + \frac{1}{g_m} + \frac{1}{g_m r_o}} \approx \frac{1}{g_m} )</td>
</tr>
<tr>
<td>( r_{out} = R_S | r_o + g_m r_o (R_S | r_o) )</td>
<td>( r_{out} = r_o (1 + g_m R_S) )</td>
</tr>
</tbody>
</table>

These configurations are characterized by near-unity current gain, low input resistance and very high output resistance. This configuration is often used as a unity-gain current buffer, but can also be used in high-speed applications where it is necessary to provide a well-defined input impedance (often 50 \( \Omega \)) and realize a moderate voltage gain. The following two examples will show how such circuits might be designed.
Example 3.16:
The common-base amplifier shown in Fig. 2.27(a) is driven by a small-signal input voltage, referenced to ground, in series with a 50 Ω source resistance $R_{source}$. The circuit should be designed so that $r_{in} = 50$ Ω and its voltage gain is unity.

As discussed earlier, the input resistance of a common-base amplifier with the base connected to a small-signal ground is approximately $1/g_m$ under the condition that $\beta >> 1$. Thus we begin with the condition that $g_m = 1/50 = 20$ mS. We then use the following expression to find the dc collector current:

$$g_m = \frac{I_C}{V_T} = \frac{\alpha_F I_E}{V_T}$$

$$\rightarrow I_E = \frac{g_m V_T}{\alpha_F} = \frac{(20 \text{ mS})(0.026 \text{ mV})}{0.99} = 525 \mu\text{A}$$

In order to realize this value of dc current, we use the following KVL equation:

$$V_B - V_{BE(on)} = I_E R_{source}$$

Recall that we can assume $V_{BE(on)}$ is a constant as long as $I_E R_{source} >> V_T$. Unfortunately this is not the case for this circuit; in fact $I_E R_{source}$ is, by design, very close to $V_T$. Thus this circuit’s biasing is not very robust. For now we will assume that we can set $V_B$ with enough precision to realize the desired biasing. In later chapters we will show how to realize better biasing for this circuit. Let us assume $V_{BE(on)} = 0.7$ V and choose $V_B = 0.726$ V.
2.3. **ONE-TRANSISTOR AMPLIFIERS**

In order to choose the appropriate value of $R_L$, we model the circuit as shown in Fig. 2.28. Using this linear circuit, we can write the following equation:

\[
v_{out} = A_i i_{in} \cdot (r_{out} | R_L)
\]

\[
i_{in} = \frac{v_{in}}{v_{in} + R_{source}} = \frac{v_{in}}{100 \, \Omega}
\]

\[
\rightarrow \frac{v_{out}}{v_{in}} = A_i \cdot \frac{r_{out} | R_L}{100 \, \Omega} 
\]  \hspace{1cm} (2.110)

For the given transistor model, we already calculated in Example 3.10 $A_i = 0.99$. For calculating $r_{out}$, we first note that $r_T >> R_{source}$; thus we can use (2.108). First, we find $r_o = \frac{V_A}{I_C} = \frac{80}{54 \, \mu A} = 146 \, k\Omega$. We now find $r_{out}$:

\[
r_{out} = 146 \, k\Omega \left[1 + \frac{(547 \, \mu A)(50 \, \Omega)}{0.026 \, V}\right]
\]

\[
= 146 \, k\Omega \left[1 + 1.05\right]
\]

\[
= 299 \, k\Omega
\]

Applying the values of $A_i$ and $r_{out}$ found above into (2.110), we find $R_L = 101 \, \Omega$, thus completing the design.
Figure 2.27: (a) Common-base voltage amplifier; (b) common-gate voltage amplifier.

Figure 2.28: Linear model of common-gate/common-base amplifier with input and output terminations.
2.4. SUMMARY OF ONE-TRANSISTOR AMPLIFIER CONFIGURATIONS

Example 3.17:
We now attempt the same design on the common-gate amplifier shown in Fig. 2.27(b).
For the input resistance, we have:

\[ r_{in} = 50 \, \Omega = \frac{1}{g_m} = \frac{V_{GS} - V_t}{2I_D} \]

Let us choose \( I_D = 2 \, mA \). This then requires \( V_{GS} - V_t = 0.2 \, V \). From this, we can write:

\[ V_G = V_{GS} + I_D R_{source} \]
\[ = 0.2 \, V + 0.4 \, V + 0.1 \, V = 0.7 \, V \]

We now calculate \( r_{out} \) using (2.109). To find the transistor \( r_o \), we use \( r_o = \frac{1}{A_D} = \frac{1}{[0.1 \, V/1(2 \, mA)]} = 5 \, k\Omega \).

\[ r_{out} = 5 \, k\Omega \left[ 1 + \frac{(2)(2 \, mA)(50 \, \Omega)}{0.2 \, V} \right] \]
\[ = 5 \, k\Omega \left[ 1 + 1 \right] \]
\[ = 10 \, k\Omega \]

In order to choose the appropriate value of \( R_L \), we once again model the circuit as shown in Fig. 2.28 and use (2.110). Applying the values of \( A_i = 1 \) and \( r_{out} \) found above into (2.110), we find \( R_L = 101 \, \Omega \), thus completing the design.

2.4 Summary of One-Transistor Amplifier Configurations

In the following table is a summary of the amplifier configurations discussed in this chapter:
### Table 2.1: Summary of one-transistor amplifier configurations.

<table>
<thead>
<tr>
<th></th>
<th>CE</th>
<th>CS</th>
<th>CC</th>
<th>CD</th>
<th>CB</th>
<th>CG</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gain</strong></td>
<td>high $A_v$</td>
<td>near-unity $A_v$</td>
<td>near-unity $A_i$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{\text{in}}$</td>
<td>high</td>
<td>$\infty$</td>
<td>very high</td>
<td>$\infty$</td>
<td>low</td>
<td></td>
</tr>
<tr>
<td>$r_{\text{out}}$</td>
<td>moderate</td>
<td>low</td>
<td>$\infty$</td>
<td>very high</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Means of power amplification</td>
<td>$A_v$</td>
<td>high $r_{\text{in}}$, low $r_{\text{out}}$</td>
<td>low $r_{\text{in}}$, high $r_{\text{out}}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>