Refinements in Determining Satellite Drag Coefficients: Method for Resolving Density Discrepancies

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The discrepancies in atmospheric densities deduced from satellites of compact and long cylindrical shapes can be used to improve our knowledge of drag coefficients. Constraints on the accommodation coefficient imposed by experiments in space and in the laboratory make it possible to resolve the discrepancies and gain information on the angular distribution of molecules reemitted from satellite surfaces. We present a sample calculation based on some limited published data. If the published nominal length-to-diameter ratio of the cylindrical satellites is the actual ratio, then we can conclude that the assumption of a diffuse angular distribution of reemitted molecules is adequate (at least near 200-km altitude), even for the long cylindrical sides where most of the molecules strike at grazing incidence. The method can be used with detailed orbital data and precise satellite shapes to infer reflection characteristics and drag coefficients for a range of altitudes and perigee velocities.

I. Introduction

The drag coefficients of satellites in the free molecular flow regime (approximately in the altitude range 150 to 500 km) usually are calculated on the basis of two assumptions: 1) The molecules that strike the surfaces of the satellite lose nearly all of their incident energy, and 2) these molecules subsequently are reemitted with a diffuse angular distribution.1-5 For satellites of compact shapes at altitudes between 159 and 290 km, these assumptions have been largely substantiated by the analyses of data from three paddlewheel satellites,6-10 the attitude-controlled S3-1 satellite,11 and the flat plate carried on the Space Shuttle Flight STS-8.12 There has always been the question of whether the foregoing assumptions are appropriate for long attitude-controlled satellites that fly like an arrow. For these cylindrical satellites, a significant portion of the drag is produced by molecules that strike the long sides at grazing incidence: In this case, the reemission could be quasispecular. The opportunity to answer this question has presented itself as a byproduct of the detailed comparison of atmospheric density measurements and models by Marcos.13 He discovered that densities deduced from three long cylindrical satellites were typically 10 to 15% below those derived from four satellites of compact shapes. He attributed this discrepancy to systematic errors in drag coefficient estimation. The present work develops methods for investigating whether assumptions 1 and 2 should be changed to improve the drag coefficient calculations and thereby reconcile the densities deduced from the two types of satellites.

II. Drag and Accommodation Coefficients of Satellites in Free Molecular Flow

The drag force $F_d$ is the component of the aerodynamic force that is antiparallel to the velocity of the center of mass of the satellite. The drag coefficient $C_d$ is defined in terms of the drag force by the relation

$$F_d = \frac{1}{2} \rho C_d A \ V_t^2$$

Here $V_t$ is the velocity of the airstream relative to the satellite, $\rho$ the atmospheric density, and $A$, a suitable reference area. Under most circumstances, the values of $V_t$ and $A$ are well known, so that the measurement of $F_d$ by an accelerometer provides knowledge of the product of $\rho$ and $C_d$.

It is convenient to calculate $C_d$ by expressing the drag force as the sum of two contributions. The first contribution is produced by the air molecules when they strike the surface. The second contribution (positive or negative) is produced as they leave the surface. This second contribution depends on the angular distribution of the reemitted molecules and the degree to which they are accommodated to the temperature of the satellite surface. The energy accommodation coefficient $\alpha$ is defined by

$$\alpha = (E_i - E_r) / (E_i - E_e)$$

where $E_i$ is the kinetic energy carried to a unit area of the surface by the incident molecules, $E_r$ the kinetic energy carried away from the unit area by the reflected molecules, and $E_e$ the kinetic energy the reflected molecules would carry away from the surface if they were reemitted at the temperature of the surface (or wall).

The value of the accommodation coefficient appropriate to a particular satellite will depend on the stream velocity, kind of incident molecule, and surface conditions. The surface conditions are influenced by many parameters. The most important is altitude, which largely determines the number of oxygen atoms striking and adsorbing on the surface. Atomic oxygen binds strongly to many surfaces and changes surface properties.15,16 An adsorbed surface layer increases the accommodation coefficient.17 It also broadens the angular distribution of the reemitted beam until, in many cases, the reemitted beam approaches the Lambert cosine law (completely diffuse reemission).18

The accommodation coefficient is a well-defined quantity when a particular gas, moving at a known velocity, strikes a particular clean crystal face of a substance (e.g., helium striking the epitaxially deposited (111) plane of gold before back-
When the years ago for gases having energies ranging from thermal energies to about 1 eV. In recent years, atomic oxygen beams have been generated with energies up to 5 eV. This is the energy with which atomic oxygen strikes satellites in low Earth orbit. Time-of-flight measurements with one of these beams confirmed what had often been seen with various gases in molecular beams at low energies: Quasispecular reemission exhibits incomplete accommodation (about 0.5 in Ref. 19), whereas diffuse (cosine) scattering shows complete accommodation. In an experiment aboard the Space Shuttle STS-8 with a perigee altitude of 225 km, a silver ring measured the scattering of oxygen atoms from a carbon surface. The scattered atoms occurred in the broad lobular pattern in Fig. 1. (The back-scattered portion of the reemitted beam was masked by the holder.) The experimenters estimated that only 2% of the scattered oxygen atoms were in a narrow lobe, whereas the rest were diffusely reemitted.

These recent results place close bounds on the accommodation coefficients that were previously deduced from three paddlewheel satellites by assuming five angular distributions measured in the laboratory at lower energies. Polar plots of the five angular distributions are shown in Fig. 2. The arrows represent the incoming particles. The recent observations eliminate all but the Schamberg diffuse model and the Alcalay and Knuth case of a broad lobular distribution seen on an old glass surface. This lobular distribution can be represented as the sum of a diffuse distribution (a sphere in three dimensions) and a cigar-shaped quasispecular component that contained about 10% of the molecules. In the Space Shuttle experiment, only 2% of the reemitted oxygen atoms were in the quasispecular lobe.

The accommodation coefficients deduced from paddlewheel satellite data by assuming five angular distributions are shown in Fig. 3, in which we shall limit our attention to the angular distribution of reemitted molecules that were used to calculate accommodation coefficients from paddlewheel satellite measurements. The figures are polar plots in the plane of incidence.

The values of \( \alpha \) calculated by assuming the diffuse and broad lobular distributions in the case of Ariel 2 at 290 km were 0.89 and 0.90, respectively. In the case of Proton 2 at 168 km, Beletsky calculated only the Maxwellian reflection coefficient, which represents the fraction of molecules diffusely reflected. Since it was 0.999, almost all of the energy was diffusely reflected, so we are safe in assuming that the accommodation coefficient was 1.00.

Paddlewheel satellites spin about the velocity vector at perigee, so that there is a torque on the canted paddles, but no net force normal to the velocity vector. On the other hand, the S3-1 satellite had an asymmetric shape and spun so that it had a measurable component of lift, as well as a (large) drag component. Ching et al. used the ratio of lift to drag to deduce that the accommodation coefficient had to lie between 0.99 and 1.00 if the observed changes in orbital inclination were to be consistent with thermospheric winds. The perigee altitude of S3-1 was 159 km.

The satellites studied by Marcos were in orbits of low and moderate eccentricity. They collected drag data at an average altitude of about 200 km. By interpolating among the values for S3-1, Proton 2, and Ariel 2, we estimate the accommodation coefficient appropriate to this altitude to be approximately \( \alpha = 0.975 \). For the calculations in the following sections, we use this value of \( \alpha \), a diffuse angular distribution, a satellite speed of 7600 m/s, a wall temperature of 300 K, a mean molecular mass of 22, and an atmospheric temperature of 920 K (an average at 200 km for the time period over which the satellites of Ref. 13 returned data). The corresponding speed ratio (satellite speed divided by the most probable molecular speed) is 9.115. In all cases, we take the reference area \( A_i \) to be the projected area of the satellite normal to the incident airstream.

III. Drag Coefficients for Satellites of Compact Shapes

It has been customary to use a drag coefficient of 2.2 for satellites of compact shapes in free molecular flow. This value of \( C_d \) is based on studies by Cook, who derived an average value which took into account the fact that satellites have many different shapes and may be tumbling in unknown ways. However, if one knows the accommodation coefficient, the angular distribution of reemitted molecules, and the detailed shape and orientation of the satellite, one can calculate \( C_d \) with greater precision. Three of the four compact satellites studied by Marcos were Atmosphere Explorers, for which the shape and orientation are known. We therefore proceed to calculate \( C_d \) for Atmosphere Explorers at 200 km, which was near the average altitude of Marcos' data.
The Atmosphere Explorer (AE) satellites had the shape of a drumlike cylinder of diameter \( D = 1.4 \) m and length \( L = 1 \) m. The spin axis, which coincided with the symmetry axis of the cylinder, was oriented normal to the airstream (orbital velocity vector). The area \( A_s \), which was chosen to be the projected area of the satellite normal to the airstream, was simply the product of length and diameter \( LD \). We calculate the drag coefficient for the cylindrical surface from three models of gas-surface interaction: the hyperthermal approximation according to Schamberg,23 the Joule gas approximation,24 and the Maxwellian velocity distribution of gas molecules superposed on the stream velocity.25,26 In all three models, we assume a completely diffuse angular distribution of reemitted molecules. An examination of the numerical results in Table 1 confirms that, for the cylindrical surface, there is little difference in the drag coefficients calculated from the hyperthermal model (which ignores the random thermal motion of the gas molecules compared to the stream velocity), the Joule gas model (which approximates the thermal motion by three pairs of orthogonal velocities superimposed on the stream velocity), and the freestream plus Maxwellian velocity distribution. The last model is the most realistic for the contribution of the incident molecules to the drag force.

We calculate the contribution of the circular flat plates (at the two ends of the cylinder) to the drag coefficient only from the freestream plus Maxwellian velocity distribution, because the other two models are inadequate for molecules striking a surface near grazing incidence, as was pointed out by Sentman.27 The result is an additional term 0.136 that must be added to the drag coefficient of the cylindrical surface to obtain a total drag coefficient for the AE satellites. The result is \( C_D = 2.35 \) at 200-km altitude.

As noted in Sec. II, the value of the accommodation coefficient \( \alpha \) in these calculations was obtained by interpolating among the S3-1, Proton 2, and Ariel 2 measurements. However, we cannot be certain that a linear interpolation is justified. To show the uncertainty in the drag coefficient caused by the uncertainty in \( \alpha \), we present in Table 2 additional drag coefficients calculated from the freestream plus Maxwellian velocity distribution, assuming wide bounding values, \( \alpha = 0.95 \) and \( \alpha = 1.00 \).

We conclude that the drag coefficient of the AE satellites at 200-km altitude is \( C_D = 2.35 \pm 0.07 \) or \(-0.11 \).

**IV. Drag Coefficients of Long Cylindrical Satellites**

Long cylindrical satellites that fly like an arrow differ from compact satellites in that a significant fraction of the drag can be caused by the random thermal motion of the ambient air: For example, the area of the cylindrical sides of such a satellite with a length-to-diameter ratio of 5 is 20 times the projected area of the front of the satellite. Even though the satellite speed can be 10 times the root-mean-square molecular speed, the area ratio magnifies the effect of molecules that strike (by virtue of their thermal motion) the long sides of the cylinder at grazing incidence.

As mentioned in Sec. III, the hyperthermal and Joule gas approximations are inappropriate for molecules that strike the surface near grazing incidence. For this reason, we omit the hyperthermal and Joule gas cases from the calculations of this section. Our knowledge of gas-surface interactions at grazing incidence is limited: Paddlewheel satellites have so far been unable to take measurements at grazing incidence, and laboratory measurements also are difficult at such angles. The satellite study of Marcos13 provides for the first time the opportunity to determine the character of gas-surface interactions in orbit at grazing incidence. The three long cylindrical satellites used in that study had a length-to-diameter ratio \( L/D \) of approximately 5. However, the exact shapes of these satellites have not been published. We therefore assume three frontal shapes for the cylinders and take the average value of the drag coefficients calculated as representative of the cylindrical satellites. Two of the shapes \( A \) and \( B \) are cylinders terminated by the frustum of a cone, with the ratio of the larger diameter \( D \) to the smaller diameter \( d \) being 3. The cone angle for the frustum \( A \) is 30 deg and that for frustum \( B \) is 45 deg. The third shape \( C \) is a cylinder terminated by a hemisphere. The drag coefficients are again calculated with the same values of orbital velocity, wall temperature, mean molecular mass, and atmospheric temperature as given in Sec. III. In all three cases, the reference area \( A_r \) is chosen to be the projected area of the satellite normal to the airstream, i.e., the circular cross section of the front of the satellite. If we assume a completely diffuse angular distribution for the reemitted molecules and the accommodation coefficient is 0.975, we obtain the drag coefficients shown in Table 3. We notice that the result for shape \( C \) is close to the average for the three shapes. Therefore, we use shape \( C \) in calculating the effect of various accommodation coefficients on \( C_D \). The results are given in Table 4.

### Table 1 Drag coefficients for the AE satellites at 200 km according to three models of gas-surface interactions, with \( \alpha = 0.975 \) and diffuse angular reemission assumed

<table>
<thead>
<tr>
<th>Surface/model</th>
<th>Hyperthermal</th>
<th>Joule gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder</td>
<td>2.184</td>
<td>2.209</td>
</tr>
<tr>
<td>Two flat plates</td>
<td></td>
<td>0.136</td>
</tr>
<tr>
<td>Total ( C_D )</td>
<td>-</td>
<td>2.353</td>
</tr>
</tbody>
</table>

### Table 2 Drag coefficients for the AE satellites at 200 km for three values of the accommodation coefficient, with diffuse angular reemission and a model of freestream plus Maxwellian velocity distribution assumed

<table>
<thead>
<tr>
<th>Surface/( \alpha )</th>
<th>( \alpha = 0.95 )</th>
<th>( \alpha = 0.975 )</th>
<th>( \alpha = 1.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder</td>
<td>2.286</td>
<td>2.217</td>
<td>2.105</td>
</tr>
<tr>
<td>Two flat plates</td>
<td>0.136</td>
<td>0.136</td>
<td>0.136</td>
</tr>
<tr>
<td>Total ( C_D )</td>
<td>2.422</td>
<td>2.353</td>
<td>2.241</td>
</tr>
</tbody>
</table>
We conclude that the average \( C_d \) for cylindrical satellites at 200-km altitude with an \( L/D \) equal to 5 is about 3.3 if the angular distribution of molecules reemitted from the long sides of the cylinder is diffuse. In case this value of the drag coefficient fails to resolve the discrepancies in the measured atmospheric densities, then a different model of reemission from the cylindrical sides could be used. For example, one might employ Sentman's formulas\(^2\) for the momentum transfer by the incident molecules and the diffuse fraction of the reemitted molecules. For the remaining fraction of reemitted molecules, one might use Schamberg's model\(^3\) with a quasi-specular angular distribution and smaller accommodation coefficient, e.g., \( \alpha = 0.5 \) from the laboratory experiments of Cross and Blais.\(^9\) Such options are included in our computer program for calculating drag coefficients.

### V. Method for Resolving Discrepancies in the Densities Deduced from the Two Types of Satellites

The drag data determine only the product of density and drag coefficient. By requiring that the densities deduced from the compact and long cylindrical satellites be independent of shape, we can gain information on the reflection characteristics of the surfaces at the particular altitudes where data are collected. Detailed orbital data and the precise shape of the satellites are required for this investigation. Although we do not have access to this information, we can present a sample calculation to illustrate the method. We use the data published in Table 2 of Ref. 13. These data are averages over time and altitude. We further average the data over the 14 models in that table to minimize the effects of atmospheric fluctuations and modeling errors. The resulting ratio of the densities deduced from the compact satellites to those deduced from the cylindrical satellites is 1.12 ±0.02. We now use this experimental ratio, together with the calculations of Secs. III and IV, to demonstrate how one can determine whether assumptions 1 and 2 need to be modified for molecules striking at grazing incidence.

The densities determined by the two types of satellites would agree if the ratio of the \( C_d \) used for the cylindrical satellites to that used for the compact shapes in Ref. 13 were reduced by 12%. In other words, if the \( C_d \) for the compact shape is correct, then the \( C_d \) for the cylindrical shape should be reduced by 12%. There is a possible physical reason why \( C_d \) for the cylindrical satellite might have to be reduced: If a significant portion of the molecules that strike the long cylindrical sides at grazing incidence are then reemitted in a quasispecular lobe rather than a diffuse pattern, the momentum transfer to the satellite and, hence, the drag would be reduced. A provision for such a distribution of reemitted molecules was made in our computer program.

### Table 3 Drag coefficients of cylindrical satellites at 200 km, with diffuse angular reemission, \( \alpha = 0.975 \), and \( L/D = 5 \) assumed

<table>
<thead>
<tr>
<th>Shape</th>
<th>Front</th>
<th>Sides</th>
<th>Total ( C_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.153</td>
<td>1.095</td>
<td>3.248</td>
</tr>
<tr>
<td>B</td>
<td>2.199</td>
<td>1.155</td>
<td>3.354</td>
</tr>
<tr>
<td>C</td>
<td>2.187</td>
<td>1.114</td>
<td>3.301</td>
</tr>
</tbody>
</table>

### Table 4 Drag coefficients of cylindrical satellites (shape C) at 200 km for different values of the accommodation coefficient, with diffuse angular reemission and \( L/D = 5 \) assumed

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>Frontal hemisphere</th>
<th>Cylindrical sides</th>
<th>Total ( C_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.950</td>
<td>2.246</td>
<td>1.114</td>
<td>3.360</td>
</tr>
<tr>
<td>0.975</td>
<td>2.187</td>
<td>1.114</td>
<td>3.301</td>
</tr>
<tr>
<td>1.00</td>
<td>2.092</td>
<td>1.114</td>
<td>3.206</td>
</tr>
</tbody>
</table>

The values of the drag coefficients used in the analysis quoted in Ref. 13 were the customary \( C_d = 2.2 \) for the AE satellites and \( C_d \) "near 3.5" for the long cylindrical satellites. The assumed ratio of cylindrical to compact drag coefficients is then \( R_c = 3.5/2.2 = 1.59 \). A reduction of 12% would give 1.40 for the correct drag coefficient ratio \( R_c \). Referring back to Secs. III and IV, we find that the assumption of diffuse reemission of molecules with an accommodation coefficient of 0.975 gives \( C_d = 2.35 \) for the AE satellites and \( C_d = 3.30 \) for the cylindrical satellites. The calculated ratio \( R_c \) of cylindrical to compact drag coefficients is then \( R_c = 3.30/2.35 = 1.40 \), in agreement with \( R_c \) for these particular satellites. Thus, the density discrepancy is resolved without the necessity of introducing different angular distributions and accommodation coefficients near grazing incidence at 200 km. This conclusion is based on several assumptions: 1) the \( L/D \) for the cylindrical satellites is exactly 5; 2) the front of the cylindrical satellites contributes 2.187 (to the drag coefficient); 3) the aft portion is a circular cylinder; and 4) the \( C_d \) used for the cylindrical satellites in Ref. 13 was exactly 3.5.

The effect of the accommodation coefficient on the calculated ratio \( R_c \) is displayed in Table 5. The uncertainties described by \( \alpha \) is about 2%. The lack of knowledge of the angular distribution of the cylinder was shown in Table 3 to yield a drag coefficient of 3.30 ±0.05. This produces an uncertainty of 1.5% in \( R_c \). Assuming the effects of \( \alpha \) and the front of the cylindrical satellites to be independent, we obtain an uncertainty of 2.5% in \( R_c \). The small uncertainty in the calculated ratio \( R_c \) and its close agreement with the corrected ratio \( R \) support the conclusion that there would be no significant departure from assumptions 1 and 2 of Sec. 1 at 200 km, provided that the assumptions of the sample calculation are correct.

The lack of knowledge of the exact ratio \( L/D \) for the cylindrical satellites is a serious limitation in our sample calculation. If \( L/D \) were 4.5, a quasispecular component could not explain the density discrepancies. If \( L/D \) were 5.5, we could resolve the density discrepancies by requiring that 17% of the molecules that strike the satellite surfaces at grazing incidence be reemitted quasispecularly with an accommodation coefficient of 0.50. (See Sec. IV.)

These results are based on average values from the published data\(^13\) that involve different satellites (in low Earth orbit) having different orbital conditions and collecting data at different times. Much better information, relating drag coefficients to altitude and perigee velocities, can be derived from the original data. On the basis of the sample calculation, subject to the conditions previously enumerated here, one would conclude that there is no need to change the usual assumptions concerning the diffuse angular distribution of reemitted molecules (even at grazing incidence) at altitudes near 200 km. However, for higher perigee altitudes (Ariel 2) or more eccentric orbits (Explorer 6), we expect a quasispecular component because of the lower accommodation. The quasispecular component can be incorporated in the analysis by using a superposition of models as described in Sec. IV. Alternative methods of incorporating a quasispecular component have been developed by Fredo and Kaplan\(^29\) and Herrero.\(^30\) These alternative models are based on laboratory measurements. To obtain more information at altitudes above 200 km, it would be desirable to fly a dedicated paddlewheel satellite experiment\(^31\) to measure the actual drag and accommodation coefficients.

### Table 5 Calculated ratio \( R_c \) of drag coefficients of the cylindrical to the compact satellites for different values of the accommodation coefficient

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0.950</th>
<th>0.975</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_c )</td>
<td>1.39</td>
<td>1.40</td>
<td>1.43</td>
</tr>
</tbody>
</table>
VI. Conclusions

The knowledge of accommodation coefficients that has accumulated over the past few decades enables us to make more refined determinations of satellite drag coefficients. If we require that atmospheric density measurements be independent of the shape of the satellite, we can further improve our knowledge of the reflection characteristics of satellite surfaces. A sample calculation based on limited published data illustrates the method: Using the nominal published values for compact and long cylindrical satellites, we arrive at the tentative conclusion that the reemission of molecules from satellite surfaces at 200 km is completely diffuse, even for molecules that strike the surfaces at grazing incidence. We have discussed the effect of uncertainties on this conclusion. When used with the original data, this method can remove the uncertainties and provide more information on reflection characteristics and drag coefficients as they depend on altitude and orbital velocity.

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References