Controlling the Breakup of a Droplet Stream

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Abstract

This paper presents both an experimental and analytical study examining the breakup of a round jet into a droplet stream. The experimental side of this work photographs the breakup of a capillary jet into uniformly sized and equally spaced droplets. The analytical portion of the work seeks to validate the linear models developed by Lord Rayleigh and Weber in the context of more complex disturbances applied to the surface of the jet. Analytical profiles are compared against the actual profiles of the jet before breakup for validation. In addition, transfer functions are proposed which allow for the inclusion of both the deterministic (experimentally driven) as well as stochastic (background noise) inputs. These transfer functions are the foundation which will be used in future work to apply dynamical feedback control strategies with the intent of controlling the droplet size and spacing in real time.

1 Introduction

Spray combustion occurs in furnaces, gas turbines, diesel engines, spark ignition engines, and rockets. The liquid fuels used in these combustion systems must be atomized in order to increase the surface area and facilitate mixing with ambient oxidizers. The size and spacing of the droplets formed greatly influences the characteristics of the resulting combustion. For example, fewer large droplets have less available surface area for chemical reaction. In spray combustion systems, however, identification and control of a particular droplet size and spacing is rarely achieved. Thus, many of these systems do not achieve their full performance potential and are also subject to increased pollutant emission.

The study of these highly complex sprays is limited. The dense multidimensional packing of droplets with chaotic velocity distributions typical in spray combustion systems makes analytical studies nearly impossible and numerical studies quite challenging. On the other extreme, isolated droplet studies do not capture the interacting environment of sprays. In order to maintain the droplet interactions but alleviate the difficulties of fully three-dimensional droplet clouds, researchers have focused on the study of rectilinear droplet streams issuing from liquid jets [1]-[3].

The study of liquid jets has a long and rich history. The early work of Lord Rayleigh [4] provided the foundation for the breakup of liquid jets. He showed in this early work that the jet is unstable to disturbances (axisymmetric) when the wavelength of the disturbance is greater than the circumference of the issuing jet. This unstable disturbance then grows exponentially with time along the surface until a drop pinches off the tip. In 1931, Weber [5] included aerodynamic terms to the theory proposed by Lord Rayleigh. The inclusion of aerodynamic terms predicted the experimentally observed shorter stream breakup. In essence, the work of Rayleigh and Weber are special cases of the work by Sterling and Sleicher [6]. They considered the linearized equations of motion and continuity along the free surface of the jet assuming a small axisymmetric disturbance that propagates along the surface. They concluded that Weber’s formulation was accurate to within 0.1% in the growth rate of the disturbance.

Subsequent to the early pioneering work in the breakup of liquid jets, much interest turned to looking at the non-linearities present. Crane et al. [7] performed studies measuring the growth rate of applied disturbances. They found that there was qualitative agreement with Rayleigh’s early work, but that more importantly, the surface wave was non-sinusoidal. Donnelly and Glaberson [8] further studied the growth rates and found that the growth rate was indeed well predicted by Rayleigh’s theory and they suggested that higher harmonics present in the system caused the non-sinusoidal behavior. This conflicting work set the foundation for extensive studies into the non-linear stability of a liquid jet [9]-[17].
Some of the more recent work in understanding the breakup of liquid jets looks at methods to control the breakup of the liquid jet in order to obtain a given droplet size. The work by Orme and Muntz [18, 19, 20], for example, looks at producing highly uniform droplet streams. They use amplitude modulation techniques to produce droplet sizes outside the range achievable using single frequency perturbation. In addition, the amplitude modulation decreases speed variations between droplets.

Since spray combustion ultimately depends on the spray creation process, controlling the droplet distribution may allow more sophisticated control of combustion than can be achieved by gas phase systems. Active breakup of the liquid jet in order to obtain a given droplet size. The work by Orme and Muntz [18, 19, 20], amplitude modulation decreases speed variations between droplets. For example, looks at producing highly uniform droplet streams. They use amplitude modulation techniques [21, 26, Kemal and Bowman [27], Fung and Yang [28], Annaswamy and Ghoniem [29], and Strayer et al. [30, 31]. In our previous work, we demonstrated that application of modern control theory to a non-premixed flame allows control of the luminosity of the flame through enhanced mixing. While the physics governing the formation of fuel droplets from bulk fluids and the enhanced mixing of gaseous fuels are different, the process of identifying the key controllable parameters for each process and identifying their effect are similar, particularly in that it is the presentation of fuel and oxidizer to the flame that governs the final combustion performance.

This paper looks at the linear theory proposed by Rayleigh and Weber and compares analytical profiles of a liquid water stream with imposed disturbances to experimental profiles. The disturbance imposed on the surface of the stream at the orifice exit consist of multiple frequency components due to either the input waveform directly or external disturbances acting on the surface of the stream. With a known disturbance and acceptance of the linear model proposed by Rayleigh, the system is deterministic. However, as stated, the forcing is not entirely deterministic. The stochastic component of the imposed disturbance propagates along the surface of the stream (assuming satisfaction of the Rayleigh criterion for unstable growth) and alters the breakup. With this in mind, the analytical portion of this work considers superposition of frequencies in an attempt to determine a realistic transfer function from the applied signal to the disturbance growing on the stream. Once a suitable transfer function is found, application of feedback control should allow for more precise control of the droplet breakup despite stochastic perturbations.

2 Theory

The growth rate, $\beta$, of an applied disturbance to a capillary jet using the theory of Weber and Rayleigh is a solution to the quadratic

$$\beta^2 + \frac{3\mu k_0^2}{\rho_0^2} \beta = \frac{\sigma}{2\rho_0^2} (1 - k_0^2) k_0^2 + \frac{\nu_0^2 \rho_a k_0^2 K_0(k_0)}{2\rho_0^2 K_1(k_0)} \tag{1}$$

where $\mu$ is the stream fluid viscosity, $k_0^* = 2\pi r_0/\lambda$ is the non-dimensional wavenumber of the disturbance, $\rho$ is the stream fluid density, $r_0$ is the orifice radius, $\sigma$ is the surface tension of the stream fluid, $v_*$ is the stream speed, $\rho_a$ is the density of the ambient fluid, and $K_n$ is the $n^{th}$ order modified Bessel function of the second kind. The disturbance applied to the stream will then grow on the surface of the stream according to

$$|r(t) - r_0|/r_0 = \pm \epsilon(t)e^{\beta t} \tag{2}$$

where $r(t)$ is the stream radius as a function of time, $r_0$ is the initial jet radius, and $\epsilon(t)$ is the initial disturbance applied to the stream. The growth of the disturbance continues until a fluid parcel is pinched off of the capillary (when $r(t) = 0$ or $\epsilon(t)e^{\beta t} = \pm 1$). This parcel is not initially spherical, but oscillates due to surface tension forces until viscous dissipation dampens out the oscillatory motion. The resulting droplet stream consists of uniform droplets with an average separation equal to the wavelength of the applied disturbance.

Figure 1 is a representative plot of the growth rate $\beta$ as a function of the non-dimensional wavenumber for water. The curve is parabolic and the maximum $\beta$(the fastest growth rate) is located at $2\pi r_0/\lambda = 0.697$. This peak describes the set of conditions resulting in the fastest breakup time (shortest capillary length) given a single frequency initial disturbance. The relatively broad bandwidth of the curve reveals that there are infinitely many potential growth rates on the surface that can theoretically determine the breakup of the jet into droplets. Hence, it will be the combination of rate and initial disturbance amplitude that pinches off the
fluid into the desired droplets that we strive to control. The fastest growth rate is a function of controllable parameters such as the speed of the jet (i.e. the pressure applied to create the capillary jet) and the radius of the jet orifice along with parameters that are nominally fixed for a particular fluid (i.e. the viscosity, density, and surface tension), though these might also be controlled through a temperature input, for example. Some have suggested that it is the fastest growth rate the ultimately determines the breakup length of the capillary jet. In fact, if we consider there to be two growth rates, one slightly less than the other, but the slower one to have an initial disturbance amplitude greater than the faster growing wave, any of the following scenarios is possible: 1) the disturbance with the faster growth rate breaks the jet up first; 2) the disturbance with the slower growth rate breaks the jet up first; 3) the two jets break up at the same time; or 4) the combination may superpose to produce a quicker pinch off and breakup. For this reason, it is not adequate to consider only the disturbance with the fastest growth rate, but rather, important to explore the relationship that multiple disturbances have on the growth and eventual formation of droplets.

The term accounting for the initial disturbance on the stream, $\epsilon(t)$, is the superposition of all disturbances present. Each of the frequency components in the disturbance has a corresponding disturbance amplitude, $a_i$, and growth rate, $\beta_i$. Thus, writing Eqn. 2 in a fashion similar to that of Huynh et al. [34] and Ashgriz and Mashayek [17] yields

$$[r(t) - r_0]/r_0 = \pm \sum_i a_i \epsilon_i(t)e^{\beta_i t}.$$  \hspace{1cm} (3)

The difficulty now lies in determining the transfer of the initial disturbance to the surface of the jet. If we assume that the forcing is a combination of the applied force (deterministic) and a white noise component (stochastic) acting on the bulk fluid housed within a chamber upstream from the exit plane orifice, then the properties of this chamber determine the disturbance transmitted through to the capillary jet. In this research, the upstream chamber consists of a piezoelectric material surrounding a cylindrical chamber. The upper portion is fixed while the lower portion with the attached orifice is free to move with the piezoelectric (Fig. 2). This arrangement is similar in function to a mass-spring-dashpot system where the orifice is considered part of the mass. The piezoelectric/steel chamber behaves like a spring with damping for small motions. A transfer function for such second-order systems is usually of the form

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$  \hspace{1cm} (4)

where $s = j\omega$, $\omega$ is the frequency (rad/s), $\zeta$ is the damping ratio, and $\omega_n$ is the undamped natural frequency.
Figure 2: Diagram of the droplet generator

In order to simplify the analysis, and realizing that the damping ratio and peak amplitude are unknowns, the transfer function is rewritten in a form non-dimensionalized about the peak amplitude. The peak amplitude occurs at the resonant frequency, \( \omega_n \). This transfer function then takes the form

\[
H(s) = \frac{B_0}{s^2 + A_1 s + A_0}
\]

where

\[
B_0 = A_1 \sqrt{A_0}
\]

for a unity peak amplitude. Substitution of Eqn. 6 in Eqn. 5 gives the expression

\[
H(s) = \frac{A_1 \sqrt{A_0}}{s^2 + A_1 s + A_0}
\]

The term \( A_1 \), from a graphical perspective controls the bandwidth of the curve as depicted in Fig 3.

The final step is to look at the frequency domain representation of Eqn 2. The transformation from the time domain to frequency domain is common in modeling dynamical systems for designing control systems. For simplicity, we will define

\[
\hat{r}(t) = \frac{-r(t) + r_0}{r_0}
\]

If we consider only positive time domain (assume \( \hat{r}(t) = 0, \forall t < 0 \)) we can use the unilateral Laplace transform equivalently with the Fourier transform

\[
\mathcal{L} (\hat{r}(t)) = \mathcal{F} (\hat{r}(t))
\]

where the Laplace transform is defined as

\[
\mathcal{L} (\hat{r}(t)) \equiv \hat{R}(s) = \int_0^\infty \hat{r}(t)e^{-st}dt.
\]

Then

\[
\int_0^\infty \hat{r}(t)e^{-st}dt = \int_0^\infty e(t)e^{-\beta t}e^{-st}dt = \int_0^\infty e(t)e^{(\beta-s)t}dt.
\]
Next, by rewriting Eqn. 11 and recalling that modulation by an exponential in the time domain is equivalent to a shift in frequency we write

\[ \int_0^\infty e(t)e^{(\beta-s)t}dt = \int_0^\infty e(t)e^{-(s-\beta)t}dt = E(s - \beta). \]  

(12)

As a result, for any periodic disturbance in time \( \epsilon(t) \), the frequency domain equivalent (Laplace transform) is shown to be

\[ \mathcal{L}\left( \hat{\epsilon}(t) \right) = \mathcal{L}\left( \epsilon(t)e^{\beta} \right) = E(s - \beta). \]  

(13)

The schematic shown in Fig. 4 shows the input-output relationship of the system. A combination of deterministic and stochastic disturbances are applied to the input of the piezoelectric transducer. A transfer function modifies this input and sends the dynamically filtered signal to the surface of the capillary stream which propagates along the stream. Now, the overall transfer function from the input to the system to the growth of the disturbance along the surface of the stream is

\[ \hat{R}(s) = H(s)E(s - \beta). \]  

(14)

This allows us to track the transmission of a disturbance through the piezoelectric onto the surface of the stream and the growth of this disturbance eventually resulting in the pinching of the jet into a droplet stream. This model will be used as an analytical comparison to the experimental results. In the future, the model will provide the foundation for the design of a control system.
3 Experimental Apparatus

At the core of the apparatus used in the experimental portion of this work is a droplet generator using a piezoelectric material. Figure 5 shows the setup used for these studies. A function generator supplies a periodic waveform that feeds the amplifier responsible for driving the piezoelectric droplet generator. In addition, the periodic waveform goes through a frequency divider where it externally gates the strobe lamp power supply. The frequency divider is necessary as the function generator operates in the kilohertz range which is beyond the capabilities of the strobe lamp/camera combination. The camera requires more light than available from a single flash of the strobe. By using the frequency divider, with a stable and reproducible set of conditions, droplets appear stationary in the camera output and multiple droplets can be imaged onto a single negative with minimal droplet-droplet variations. Furthermore, a digital delay box allows slight phase shifts between the incoming function generated signal and the strobe firing. As the delay is changed, the movement of the droplets are imaged on the camera/screen. The supply to the droplet generator is a water filled reservoir which is pressurized to maintain constant flow rate to the generator despite small changes in reservoir head. When verification of the droplet frequency is required, a HeNe Laser focused along a stable portion of stream scatters light onto a PMT as droplet pass through.

4 Results

Figure 6 is a magnified photograph of the capillary stream showing the growth of the disturbance along the surface as well as the formation of a stable stream of downstream droplets. Figure 7 shows a portion of the capillary stream and the pinching off of droplets using water with a $75\mu m$ orifice at a square wave forcing frequency of 9.73 kHz. The images are separated by $10\mu s$ and show a time history of the breakup process. It is apparent from these photos that the actual breakup is more complicated than predicted with the Rayleigh/Weber model assuming single frequency perturbation (i.e. the snapping back of the tip as the fluid pinches off and the wave that propagates back). However, an analytical model with just a single input frequency appears to capture much of the detail present in these operating conditions (Fig. 8) when we look at
Figure 6: Growth of sinusoidal disturbance on a water capillary stream (75μm orifice, f=9.73 kHz, p=13.25 psi, 32X).

Figure 7: Growth of sinusoidal disturbance on a water capillary stream (75μm orifice, f=9.73 kHz, p=13.25 psi, Δt = 10μs).
the time when the droplet pinches off the capillary stream. Under different operating conditions, however, the breakup process appears different. In Fig. 9, the original photograph clearly shows that the breakup length oscillates. Figure 10 shows this phenomenon zoomed in. This was first observed as a series of long exposure photographs showing what appears to be a capillary jet surrounded by a droplet. After many more experiments using the delay box to step through the breakup process, the phenomenon is attributable to a periodic breakup length. The Rayleigh breakup model does not reveal this behavior. It was originally hypothesized that perhaps this unstable breakup length was a function of the square wave forcing used. An analysis of a square wave via a Fourier series representation shows the forcing to be of the form

$$
\epsilon(t) = \frac{F}{2} + \frac{2F}{\pi} \sin(t) + \frac{2F}{3\pi} \sin(3t) + \frac{2F}{5\pi} \sin(5t) + \cdots
$$

(15)

where $F$ is the amplitude of the forcing. As it was shown previously, growth rate $\beta$ is a function of the wavenumber $2\pi r_0/\lambda$. If the velocity of the stream is sufficiently low, then the fundamental frequency is the only component within the square wave that satisfies the Rayleigh criterion for unstable disturbance growth. If the velocity of the stream exceeds some value then both the fundamental and the first harmonic satisfy the Rayleigh criterion. The relative amplitudes of the frequency components are given by Eqn. 15. Thus, driving the system with a square wave at a given frequency with two different jet velocities can, for example, show breakup characterized by the fundamental frequency for the low velocity and the harmonic at the higher velocity. Also, given both a sine wave and square wave of equivalent frequencies at the same jet velocity, the harmonic present in the square wave may cause breakup at the harmonic frequency component as shown in Fig. 11. However, a sensitivity analysis of the breakup length to changes in amplitude of disturbance shows that in order to see a 10% fluctuation in breakup time requires approximately a 50% fluctuation in initial amplitude (see Fig 12.) To have white noise at other frequencies account for these fluctuations would require amplitudes within an order of magnitude of the driving amplitude. Fig. 12.

5 Conclusions and Future Work

When a droplet stream issues from an orifice, any external disturbances impose on the surface of the stream will grow in an exponential manner until a fluid parcel pinches off. The original work by Rayleigh, with
Figure 9: Photograph of the capillary stream along with what appears to be a fully separated droplet with a portion of the stream penetrating it.

Figure 10: Zoomed in photograph of the breakup portion along the capillary stream showing the appearance of the stream inside the droplet.
Figure 11: Analytical comparison of sinusoidal (left) and square wave (right) forcing under the same conditions showing the higher frequency harmonic component contribution to breakup. Note that the aspect ratio is exaggerated in horizontal dimension for comparison.

Figure 12: Plot of the disturbance amplitude required for a range of wavenumbers to achieve a desired breakup time. Plots show the amplitude required for breakup times of 0.7ms, 0.8ms, and 0.9ms.
modifications proposed by Weber, reasonably predict the behavior of this breakup process. More recent work implementing sophisticated models using the equations of continuity and momentum provide more insight into this breakup and reveal some of the experimentally observed nonlinearities. However, as has been shown in previous work, complex systems with low order linear models can be capably controlled. This work shows that using the linear theory and considering both deterministic and stochastic disturbances, a model can be developed which predicts the breakup behavior.

The future work will look to more formally matching this behavior and subsequently consider dynamical feedback control strategies that can be used to reduce the unsteady breakup and thereby produce a more uniform breakup and hence more stable array of droplets. A sensitivity analysis of breakup to perturbations in stream velocity will be performed. For fundamental flame studies, having a more uniform burning droplet array reduces experimental uncertainty and allows for more quantitative results. Future application of control to more complex spray systems may help to achieve more uniform droplet size and spacing distributions allowing for improved combustion performance.

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References


