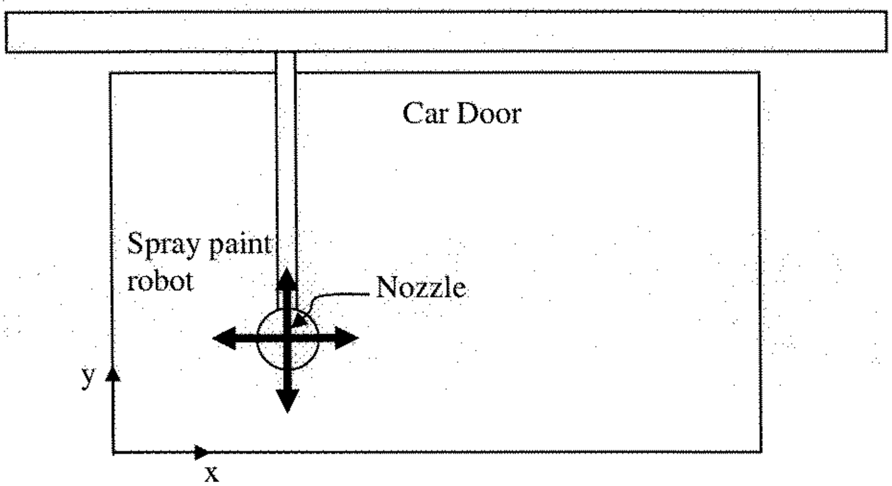
Name:
Lab Day and Time:
Circle your TA: Laura Steve Albert James

MAE 106 Mechanical Systems Laboratory
Spring 2007 Design Exam

You are a control engineer working with a team of engineers for an automobile manufacturer. Your team's goal is to design a robotic spray painting system. The robotic spray painter must paint the rectangular door panels of cars.



The paint engineer wants the paint to be sprayed on with a sinusoidal sweeping motion in the horizontal (x) direction. You must design two controllers: one that controls the motor for the horizontal movement of the robot, and one that controls the motor for the vertical movement of the robot.

You are given the above schematic and the following additional information to work with:

- The door panels are 2 meters wide and 1 meter tall.
- The nozzle creates a spray width at the door of 0.1 m diameter.
- The robot arm behaves like a mass (M = 10 kg) with damping 100 Ns/m.
- Each motor is controlled by a DC brushed motor attached to a lead screw, and powered by a current amplifier. A one volt input into the current amplifier gives 100 N of motor force to move the robot.
- You have access to the voltage signal from two potentiometers that measure the horizontal and vertical position of the robot arm. The pots' voltages are zero at x = 0 m and y = 0 m (lower left corner of door), and have a calibration coefficient of 1 V/m.
- Your vertical robot controller will receive a 0-10 Volt step change, and should move from the bottom of the door to the top of the door.
- Your horizontal robot controller will receive a sinusoidal input signal (that you will specify) that varies from 0 meters to 2 meters at 1 Hz.
- Your manager wants you to use a minimum amount of control circuitry for this system, so he says that you must try to use a proportional controller (i.e. no derivative or integral terms) in both directions. You are not sure this is possible, but you will check.
- The safety engineer wants the robot to have the smallest proportional gain possible.

Name: SOLUTION DESIGN EXAM 2007

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Design a controller to control the robotic spray painter. For parts 1-3, express answers in MKS units.

10 1) Show a control law for the robotic conveyor belt that relates the horizontal motor force (F_x) to the desired (x_d) and measured robot position (x), and a control law that relates the vertical motor force (F_y) to the desired (y_d) and measured robot position (y). Use the symbol K_{px} and K_{py} for the proportional gains in the horizontal and vertical directions.

Horizontal control law	Vertical control law
(5) $F_x = -K_{px}(x - x_d)$	(5) $F_y = -K_{py}(y - y_d)$

20 2) Specify the sinusoidal input x_d for controlling the horizontal sweeping of the robot:

$x_d(t) = 1 - \cos(2\pi t)$

$= 1 - \sin(2\pi t + \pi/2)$

$\omega = 2\pi f$ $f = 1$ Hz correct frequency only ⇒ 5pts

$t = 0$ $x_d(t) = 0$

$1 + \sin(2\pi t)$ ⟶ starts at $x = 1$ m, 10 pts partial credit

40 3) Choose gain values for the control laws. State units.

Horizontal control law

K_{px} = 39,478 N/m

$4000\pi^2$ N/m

(10) $\omega_c > 2\pi(1)$ rad/sec for good tracking

(10) use $K_p = \omega_n^2 M$

Desired bandwidth for horizontal direction:

$\omega_c = 10 \times 2\pi(1) = 20\pi$ $\quad \omega = 2\pi f = 2\pi(1\text{ Hz})$

$= 62.8$ rad/sec

Reason: Factor of 10 is arbitrary, but should be some multiple of the input frequency (1 Hz) for good tracking

More work for finding K_{px}:

For 2nd order system

$\omega_n = \sqrt{\frac{K_p}{m}} \approx$ cutoff frequency

$K_p = \omega_n^2 M = (20\pi)^2 10 = 4000\pi^2 \frac{N}{m} \approx 12{,}566 \frac{N}{m}$

for this K_p, $\zeta = \frac{B}{2\sqrt{K_p M}} = \frac{100 \text{ Ns/m}}{2\sqrt{4000\pi^2 10}} = 0.025$

⇒ very underdamped

Vertical control law

K_{py} = 250 N/m

(10) $\zeta \geq 1$

(10) $K_p = \frac{1}{M}\left(\frac{B}{2\zeta}\right)^2$

+10 Extra Credit if realize $\zeta = 2$ or 3

Desired damping ratio for vertical direction:

ζ = 1 or greater

Reason: Move slowly enough with no overshoot (see extra page of solution)

More work for finding K_{py}:

$\zeta = \frac{B}{2\sqrt{K_p M}} \Rightarrow K_p = \frac{1}{M}\left(\frac{B}{2\zeta}\right)^2 = \frac{1}{10}\left(\frac{100}{2(1)}\right)^2$

$= \frac{1}{10}(50)^2 = 250$

Note, for horizontal direction,

Say we choose $\zeta = 1 \Rightarrow K_p = 250$ N/m (from above)

then $\omega_n = \sqrt{\frac{K_p}{M}} = \sqrt{\frac{250}{10}} = \sqrt{25} = 5$ rad/sec

$f_c = \frac{\omega_n}{2\pi} < 1$ Hz → Not good tracking

10 4) Comment on whether it would be better to include a derivative term in your controller.

In horizontal direction, yes, to avoid resonance from underdamped system

20 5) What gain value should the proportional gain stage of your op amp circuit have, if you were to design an op amp circuit to implement the horizontal controller? Express in terms of the symbol K_{px}.

+10 pts if recognize need calibration for pot & motor

$x \longrightarrow [K_{px}] \longrightarrow F$

$x \longrightarrow [1 \text{ V/m}] \longrightarrow [G] \longrightarrow [100 \text{ N/V}] \longrightarrow F$

$F = 100 G x = K_{px} x$

$G = \frac{K_{px}}{100}$

Design Exam 2007 – more detailed solution, following reasoning of Steve Spencer

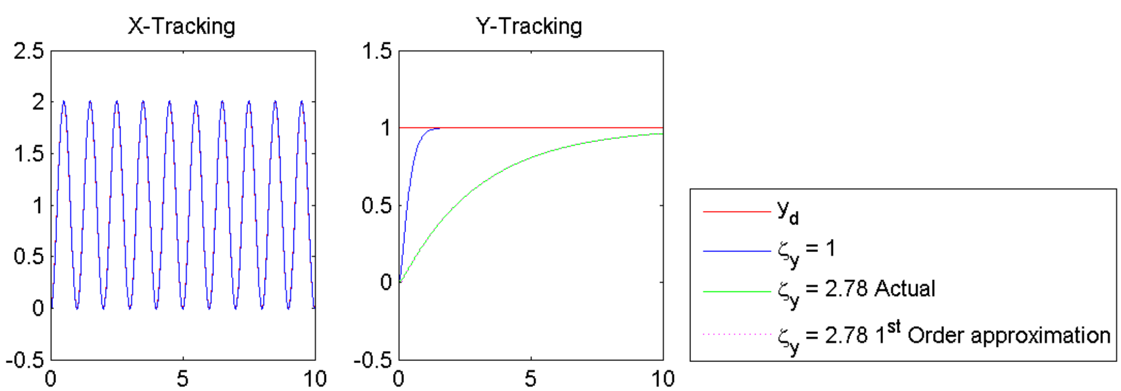
A key goal of the spray-painting robot is that the paint completely cover the door. If the robot moves too quickly in the vertical direction, then parts of the door will be left unpainted (see bottom left panel in below figure for an example showing spray path on door). The door height is 1 m, and, thus, when the robot has traveled one time constant, it will have traveled 0.63 m. Since the effective "brush" diameter of the spray painter is given to be 0.1 m, we desired at least 3 back-and-forth motions (6 sweeps total) to be achieved after 1 time constant (because 0.63/.1 = 6) in order to cover the door with a 0.1 m "brush". Now, the back-and-forth left-right motion takes 1 second (1 Hz oscillation), so 3 back-and-forth motions = 3 seconds; i.e. our vertical time constant should be 3 seconds. What should Kp be?

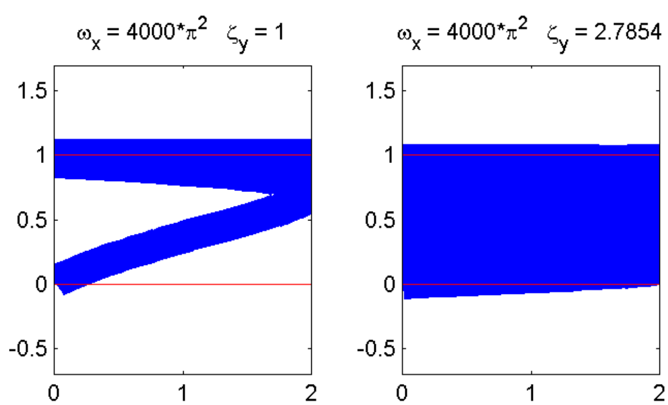
Note that an overdamped second order system can be approximated as a first order system (see top right panel in below figure for graphical example) dominated by the "slower" pole, which is the pole closer to the jω axis. The poles for the 2nd order mass, spring, damper system are:

$$s = \frac{-B \pm \sqrt{B^2 - 4MK_p}}{2M}$$ and the slower pole is: $$s_s = \frac{-B + \sqrt{B^2 - 4MK_p}}{2M}$$

We desire $s_s = \frac{-1}{\tau} = \frac{-1}{3\sec}$. Solving for K_p we find: $$K_p = \frac{B^2 - (B - 2M/\tau)^2}{4M} = 32.2 \ N/m$$

which gives a damping ratio ζ_y=2.78, i.e. an overdamped system that moves mores slowly (0.67 meters in 3 seconds). This controller does a better job of painting the door than a critically damped controller (compare bottom two panels in figure, which show the spray paint pattern for the two damping ratios). The paint distribution is very uneven though, with the top of the door receiving more paint as the robot slows down. So another strategy would be to make the robot move in .1 m steps vertically, holding the vertical position constant during each paint sweep.





Bottom two plots show spray paint pattern on door for two different Kp gains (critically damped – left plot, over-damped, right plot)