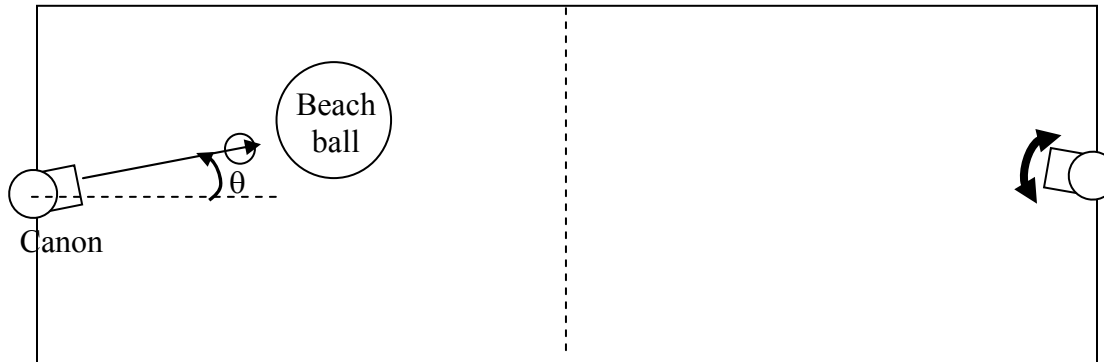


**MAE 106 Mechanical Systems Laboratory
Design Exam 2009**

In this exam, you will design a controller for a robotically controlled air canon. Assume the canon has rotational inertia $J = 1 \text{ Nms}^2/\text{rad}$, and the angle of the canon is θ , as shown below.



TOP VIEW

First you will design a proportional-derivative position controller to control the canon angle, of the form:

$$\tau = -K_p(\theta - \theta_d) - K_d\dot{\theta}$$

where τ is the motor torque in Newton-Meters, and θ is expressed in radians. You wish to keep K_p as low as possible. You also desire to have the canon move as quickly as possible when you give it a step change in θ_d . Let's assume you are designing the controller for a 90 degree step change in θ_d , when the target ball has a radius r and is at distance d away from the canon, where $r/d = 0.0787$.

1. What tolerance band should you choose for the settling time?
 A. 1% B. 10% **C. 5%** D. 2% E. 3%

At a distance d , half of the ball subtends a angular window of size $= \text{atan}(r/d) = 4.5$ degrees. For a 90 degree step, this is a percentage of $100 \cdot 4.5/90 = 5\%$. If our controller settles to within 5% of the step size, it will hit the ball when it shoots.

2. Given the chart to the right, where T is the time constant of the controlled canon, what would be the best value for the damping ratio?

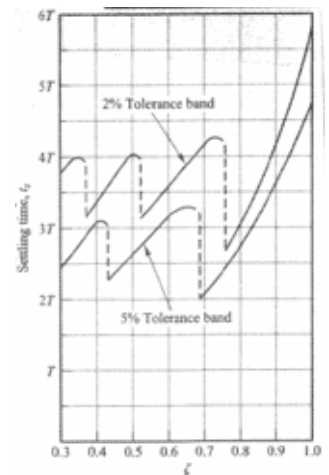
- A. 0.68** B. 0.43 C. 0.76 D. 0.72 E. 1.0

Using the 5% settling criteria, the minum settling time comes at $\zeta = 0.68$.

3. Assume that you chose the damping ratio to be 0.5, and that you would like the canon to move to the target in 0.4 seconds with a 2% tolerance band. Which value below is closest to the correct value for K_p ?

- A. 0.234 B. 0.4 C. 4 D. 40 **E. 400**

For a 2% tolerance band with a damping ratio of 0.5, the settling time is $4T$ (see graph at right). Thus we desire $4T = 0.4$, which means the time constant $T = 0.1$ sec. Now the time constant is given by $1/(\zeta\omega_n)$. Plugging in for ω_n , gives $K_p = (1/\zeta T)^2 J = (1/(.5 \cdot .1))^2 \cdot 1 = 400$



4. What are the units of K_p ?
 A. Nm/deg **B. Nm/rad** C. Nms² D. N/m*rad E. N

5. Assume that you chose K_p to be 4 and the damping ratio to be 0.8. Which value is closest to the correct value of K_d ?

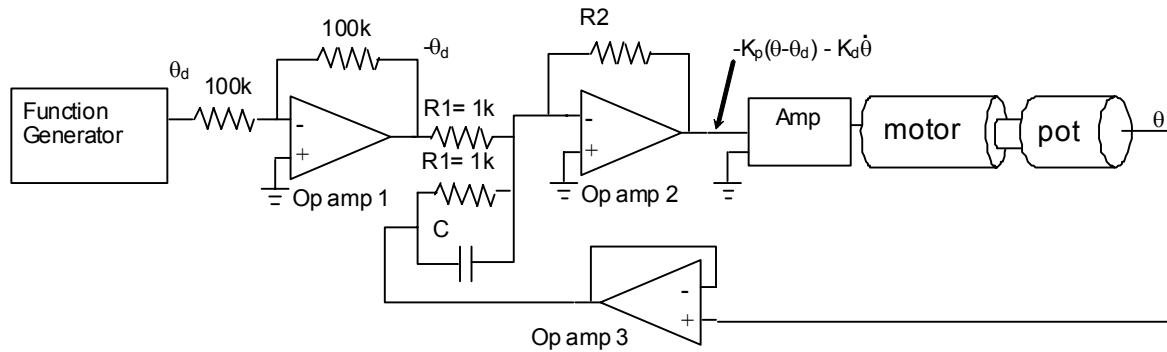
- A. 1.0 B. 2.4 C. 6.4 **D. 3.2** E. 4.8

$\zeta = K_d / (2 \cdot \sqrt{K_p J})$ so $K_d = 2 \cdot \sqrt{K_p J} \cdot \zeta = 2 \cdot \sqrt{4 \cdot 1} \cdot .8 = 3.2$

6. What are the units of K_v ?

- A. Nsm/deg B. **Nms/rad** C. Nms² D. Ns/m*rad E. N

7. Below is a circuit for implementing the controller. The canon angle is sensed by a pot. The desired canon angle is given by a voltage with the same range and scaling as the motor pot. When $\theta = 0$ degrees, the pot voltage is 0 volts, and when $\theta = 1$ radian, the pot voltage is 1 volt. If you want K_p to be 4, which value of R_2 will give you the value closest to what you desire?



- A. 1Kohm B. **4Kohm** C. 8Kohm D. 2Kohm E. 16 Kohm

Since the pot calibration has unity gain (1 volt/1 radian), the proportional gain is just R_2/R_1 so R_2 should be 4 Kohm. Note that this assumes that the amp/motor gives a torque equal to 1 NM/1 volt input.

8. You would like to use the canon in a scanning mode as well, in which it tracks a sinusoid in position from $\theta = -90$ degrees to $\theta = 90$ degrees; i.e. $\theta_d = 90 \cdot \sin(2\pi \cdot f \cdot t)$ degrees. You would like the scanning frequency f to be as fast as possible, but you know that the canon acts like a low pass filter, so, for higher scanning frequencies, it will not detect the ball in your own corners. As an experiment, you try a scanning frequency of 0.95 Hz with $K_p = 4000$ and $K_d = 4000$. How much of the corners (in degrees) will the canon not reach? Find the closest answer:

- A. 10 degrees B. 20 degrees C. 40 degrees D. 50 degrees E. **60 degrees**

If you put a sine wave in, you get a sine wave out, scaled and shifted. We need to find the scaling, which means finding the

magnitude of the transfer function. The transfer function is $G(s) = \frac{K_p}{Js^2 + K_d s + K_p}$, so

$$|G(j\omega)| = \frac{K_p}{\sqrt{(K_p - J\omega^2)^2 + (K_d\omega)^2}} \quad \text{now } \omega = 2\pi f = 6 \text{ rad/s so } |G(j\omega)| = .16. \quad \text{Since } \theta_d \text{ has an amplitude of 90 degrees, the}$$

robot will actually move peak to peak $|G(j\omega)| \cdot 90 \text{ degrees} = 14.8 \text{ degrees}$. That means it will not reach the last 75.2 degrees of the corner (bad news – need to improve that design by increasing the p and d gains!!)

9. The canon has an infrared sensor on it that outputs 5 volts when the ball is in front of the canon. Say you measure a sudden change in voltage from the sensor to a 5 volt output at $t = 1$ second, for a scanning rate f of 0.95 Hz and a desired scanning position of $\theta_d = 90 \cdot \sin(2\pi \cdot f \cdot t)$, with $K_p = 4000$ and $K_d = 4000$. At what angle θ are you detecting the ball? Find the closest answer:

- A. 0 degrees B. **-15 degrees** C. +15 degrees D. +50 degrees E. -50 degrees

You can solve this problem by finding $\theta = 90 \cdot |G(j\omega)| \cdot \sin(2\pi f t + \phi)$ where the transfer function phase shift is

$$\phi = -\tan^{-1}\left(\frac{K_d\omega}{K_p - J\omega^2}\right)$$

Plugging in for everything and finding θ gives -14.6 degrees. Note that the desired angle at this time is -27.8 degrees, so the tracking is fairly poor.