

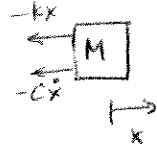
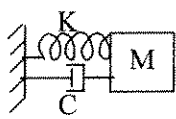
MAE 106 Mechanical Systems Laboratory
Winter 2003 Design Exam

Answer
sheet

Part 1: In-Class Problems – NOTE: You must show your work to get full credit. You will not full credit if you just write the answer – you need to show how you got the answer.

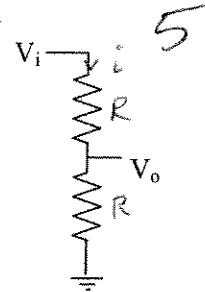
30/30

1. Write the differential equation that describes the following system:



$$M\ddot{x} = -kx - c\dot{x}$$

$$M\ddot{x} + c\dot{x} + kx = 0$$



2. Using Ohm's and Kirchoff's laws, find V_o for this circuit (Assume both resistors have value R). Note that you will NOT get credit if you simply write the answer – you must derive the answer.

$$i = \frac{V_i}{2R} \quad V_o = iR = \frac{R}{2R} V_i = \frac{1}{2} V_i$$

3. Given $\dot{x} + Ax = A$ $x(0) = 0$ Find $x(t)$.

$$G: \dot{x} = -Ax \quad x = Ce^{-At}$$

$$P: x_p = 1 \quad Ax_p = A$$

$$T: x = Ce^{-At} + 1 \quad x(0) = 0 \Rightarrow C = -1 \Rightarrow x = (1 - e^{-At})$$

4. What is $s_1 s_2$ if $s_1 = 2+j$, $s_2 = 2-j$?

$$(2+j)(2-j)$$

$$4 + 2j - 2j - j^2$$

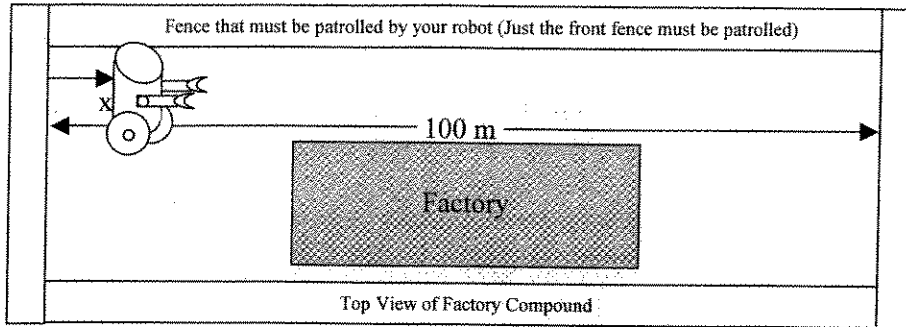
$$5$$

5. Give the amount by which a 100 Hz sinusoid input at V_i would be scaled and phase shifted in creating the output V_o for the following circuit. Assume $R = 1K\Omega$ and $C = 1 \mu F$.

$V_o = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_i$
 $\frac{V_o}{V_i} = \frac{1}{1 + RCs} = G(s)$
 $RC = (10^3)(10^{-6}) = 10^{-3}$
 $\omega = 2\pi f = (2\pi)(100) = 628 \frac{\text{rad}}{\text{sec}}$
 $G(j\omega) = \frac{1}{1 + RCj\omega} = \frac{1}{\sqrt{1 + (10^{-3})(628)^2}}$
 $2 |G(j\omega)| = \frac{1}{\sqrt{1 + (RC\omega)^2}} = \frac{1}{\sqrt{1 + (0.628)^2}} = \frac{1}{\sqrt{1.394}} = \frac{1}{1.18} = 0.85$
 $2 \phi_{G(j\omega)} = 0 - \tan^{-1} RC\omega = -\tan^{-1} RC\omega = -\tan^{-1}(10^{-3})(628) = -\tan^{-1}(0.628) = -0.56 \text{ rad}$

Part 2: Control Design Exam

You are a control engineer working for the security department of a weapons factory. Your task is to design a mobile robot that can patrol the fence in front of the factory. The robot should also be able to move quickly to confront any intruders that are attempting to penetrate the fence.



You are given the following information and equipment to work with:

- The fence that must be patrolled is 100 m wide.
- The robot behaves as a mass $M = 100$ kg with some damping $B = 1$ N/s. $\text{N}^{\text{s}}/\text{m}$
- The robot is controlled by a DC brushed motor and a linear current amplifier. A one volt input into the current amplifier gives 1 Nm of motor torque.
- You have access to the voltage signal from a potentiometer that measures the rotation of the robot wheels, and thus the position (x) of the robot along the fence. The pot voltage is zero at $x = 0$ m, and 1 volt at $x = 100$ m.
- The robot must be able to operate in two modes with the same controller: patrol mode and attack mode.
- In patrol mode, the robot must move back and forth along the fence at 0.1 Hz. The controller receives a sinusoidal command voltage from the command center in the factory. The voltage varies between 0 and 1 volt at 0.1 Hz telling it to patrol the fence in this periodic fashion.
- In attack mode, the robot must move within 1 sec to the position of an identified intruder. The intruder is identified by a camera, which radios a command voltage to the robot. A command voltage of 0 corresponds to an intruder at left end of the fence, and a command voltage of 1 corresponds to the right end of the fence. Voltages between 0 and 1 correspond to other locations along the fence. When an intruder is identified, the command voltage changes in step function fashion.
- You must implement your controller using op-amps, resistors, and capacitors.
- The radius r of the wheels of the robot is 0.1 m.

Design an analog circuit to control the security robot. You will get full credit if you:

- 1) Show a control law for the robotic conveyor belt that relates the motor torque to the desired (x_d) and measured robot position along the fence (x).
- 2) Write the differential equation that describes the dynamics of the *controlled* robotic system.
- 3) Choose gain values for your control law. State units. Make sure to consider both patrol and attack modes, and remember that one set of gain values must work for both modes.
- 4) Briefly explain why you chose the gains you did.
- 5) Draw an op-amp circuit that can implement your controller. Label what the inputs and outputs of your circuit should be connected to.
- 6) Choose appropriate values for the resistors and capacitors in your circuit.

PLEASE ENTER YOUR ANSWERS ON THE ANSWER SHEET

Part 2: Answer Sheet – Put your answers in the boxes, and show your work to the right of the boxes.

100 pts

1) Answer:

$$\tau = -K_p(x-x_d) - K_D \dot{x}$$

$$\tau = -r K_p(x-x_d) - r K_D \dot{x}$$

sign error = -5, τ

2) Answer:

$$M\ddot{x} + C\dot{x} + Kx = Kx_d$$

$$C > B + \frac{K_D}{r} \quad K = \frac{K_P}{r}$$

+10 if just dimensions

+5 if show controlled system clearly

$$M\ddot{x} + B\dot{x} = \frac{\tau}{r} = \frac{-K_P}{r}(x-x_d) - \frac{K_D}{r}\dot{x}$$

$$M\ddot{x} + \underbrace{(B + \frac{K_D}{r})}_C \dot{x} + \underbrace{\frac{K_P}{r}}_K x = \frac{K_P}{r} x_d$$

3) Answer:

ω_n 5 K_P 397 K_D 126 ω_n 5
 6.3% 1 397 ω_n 126 ω_n 5
 4% 1 100 ω_n 79.9 ω_n 5
 -2 units, -2 units

Parallel Mode: $\frac{x}{x_d} = \frac{K}{ms^2 + cs + K}$ $\omega_n = \sqrt{\frac{K}{m}} = 63$

Set $\omega_n = 2\pi f$
 $\Rightarrow 2\pi(6.3) = 100$
 $\Rightarrow 6.3 \text{ rad/s}$

Cut-off frequency $\omega_c = \frac{1}{C}$

4) Answer:

$\zeta = 1$ desirable for no overshoot/resonance
 Attack mode, $\frac{1}{2\omega_n} = 1 \text{ sec} \Rightarrow \omega_n = 0.5 \text{ rad/sec}$
 Patrol mode, $\omega_n = (10 \times) (2\pi \cdot 1 \text{ Hz}) \Rightarrow \omega_n = 6.3 \text{ rad/sec}$
 choose bandwidth to be 10x patrol freq

-10 if no consideration of patrol mode

Set $\zeta = 1 = \frac{2\sqrt{mK}}{B + \frac{K_D}{r}}$

$$= \frac{2\sqrt{100 \frac{K_P}{r}}}{B + \frac{K_D}{r}}$$

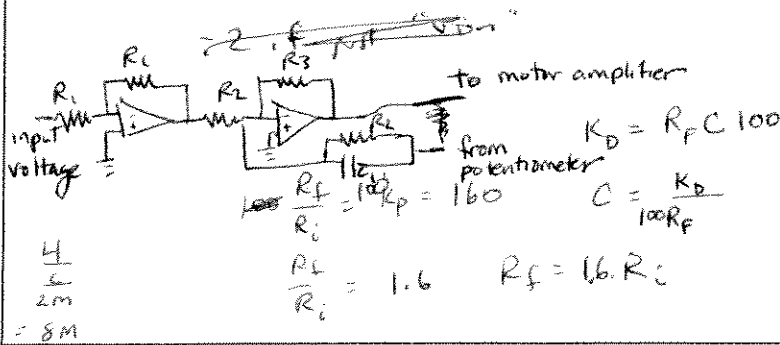
$$= \frac{2\sqrt{100 \text{ kg} \frac{397 \text{ kg/m}}{r}}}{B + \frac{K_D}{r}}$$

$$= \frac{2\sqrt{100 \text{ kg} \frac{397 \text{ kg/m}}{1 \text{ m}}}}{B + \frac{K_D}{1 \text{ m}}}$$

Attack mode: $\omega_n = 0.5 \text{ rad/sec}$
 $63 \text{ rad/sec} = \sqrt{\frac{K_P}{10 \text{ kg/m}}}$
 $K_P = 397 \frac{\text{kg/m}}{\text{s}^2}$
 check $N = \frac{\text{kg/m}}{\text{s}^2}$
 $Nm = \frac{\text{kg/m}^2}{\text{s}^2}$
 $\frac{Nm}{m} = N \cdot \frac{\text{kg/m}}{\text{s}^2}$ ✓

5) Answer:

-2 if motor instead of motor amplifier?



$$= \frac{1 \frac{Ns}{m} + 10 \frac{K_D}{m}}{2 \sqrt{3470 \frac{K_P^2}{s^2}}}$$

$$= \frac{1 \frac{Ns}{m} + 10 \frac{K_D}{m}}{126 \frac{1}{s}}$$

$$126 \frac{1}{s} = 1 \frac{Ns}{m} = 10 K_D \frac{1}{m}$$

$$K_D = 125 \frac{\text{kg/m}}{\text{s}}$$

6) Answer:

$$R_i = 100 \Omega$$

$$R_f = 1.6 \text{ M}\Omega$$

$$C = 5 \text{ mF}$$

$$K_P = \frac{R_f}{R_i} A \quad \text{where } \tau = K_P(x-x_d) - K_D \dot{x}$$

$$K_D = \frac{R_f}{R_i} A$$

$$\tau = A(-K_P(x-x_d) - K_D \dot{x})$$

$$\tau = 100(-K_P(x-x_d) - K_D \dot{x})$$

Attack mode

choose

$$\zeta = 1, \quad \frac{1}{2\omega_n} = \frac{1}{4} = 1 \text{ sec} \Rightarrow \omega_n = 0.25 \text{ rad/sec}$$

for patrol mode, we chose $\omega_n = 6.3 \text{ rad/sec}$, which

is more stringent and makes K_P bigger \Rightarrow

gains for patrol mode will work for attack mode

$$\frac{4}{1 \text{ m}} = \frac{4}{2 \text{ m}} \sqrt{\frac{K}{m}}$$

$$= \frac{4}{8 \text{ m}}$$

$$b = R_P C A$$

$$= \frac{K_D}{R_P A}$$

$$= \frac{10 \text{ kg/m}}{1.6 \times 10^6 \text{ kg/m}} = 6.25 \times 10^{-6} \text{ s}$$

$$1 \text{ V} = 100 \text{ m}$$

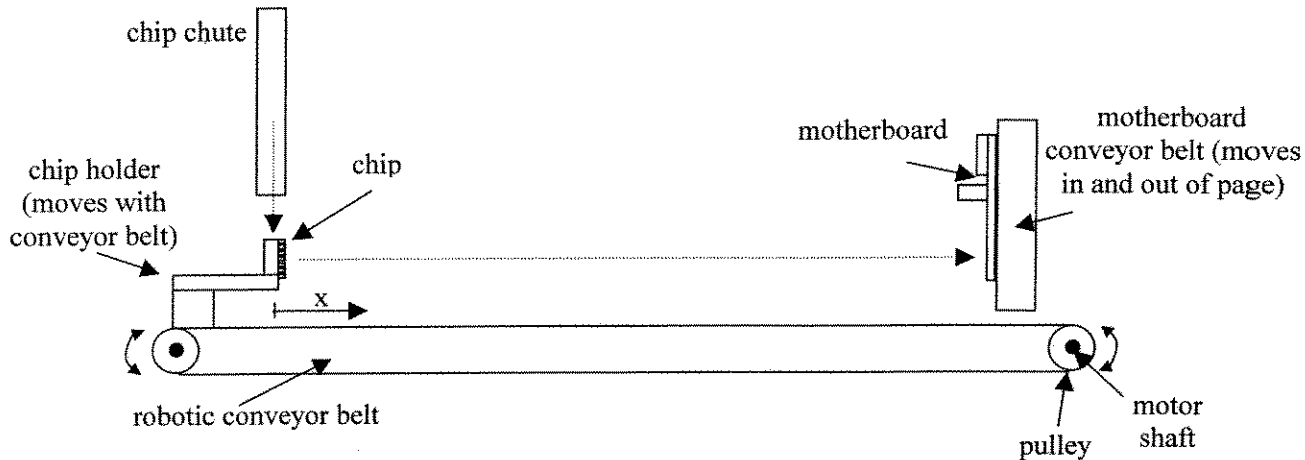
$$1 \text{ V} = 1 \text{ Nm}$$

let $R_f = 1.6 \text{ M}\Omega$
 $R_i = 100 \Omega$

$$C = \frac{10 \text{ kg/m}}{1.6 \times 10^6 \text{ kg/m} \times 6.25 \times 10^{-6} \text{ s}} = 1 \text{ mF}$$

MAE 106 Mechanical Systems Laboratory
Winter 2002 Design Exam

You are a control engineer working for a mass-manufacturer of computers. Your task is to design a robotic conveyor belt for the motherboard assembly line. The conveyor belt should catch a microprocessor chip that falls from a chute, and plug it into a motherboard that arrives on another conveyor belt:



You are given the following information and equipment to work with:

- the chip chute is at $x = 0$ m and motherboard is at $x = 1$ m
- chips are ejected out of the chip chute onto the chip holder once per second. Likewise, a new motherboard arrives on the motherboard conveyor belt once per second. The delay between arrival of a new chip and arrival of a new motherboard is 0.5 seconds. The chip holder must hold stationary for 0.1 seconds at the chip chute to catch the chip, and at the motherboard to insert the chip. Thus, your robotic conveyor belt must move between the chip chute and the motherboard conveyor belt in 0.4 seconds.
- the conveyor belt behaves like a linear mass ($M = 1$ kg) and a linear damper ($B = 1$ N/s).
- the conveyor belt is controlled by a DC brushed motor with a linear current amplifier. A one volt input into the current amplifier gives 1 Nm of motor torque.
- you have access to the voltage signal from a potentiometer that measures the rotation of the conveyor belt pulleys, and thus the position (x) of the chip holder along the conveyor belt. The pot voltage is zero at $x = 0$ m, and 1 volt at $x = 1$ m.
- your controller receives a 0-1 V square wave voltage input telling it to move from the chip chute to the motherboard. When the input voltage is 0 V the conveyor belt should move to the chip chute. When the input voltage is 1 V the conveyor belt should move to the motherboard.
- you must implement your controller using op-amps, resistors, and capacitors.
- the radius of the pulley that drives the conveyor belt is $r = .1$ m

Design an analog circuit to control the robotic conveyor. *You will get full credit if you:*

- 1) Show a control law for the robotic conveyor belt that relates the motor torque to the desired (x_d) and measured chip holder position (x).
- 2) Choose gain values for your control law. State units.
- 3) Explain why you chose the gains you did.
- 4) Draw an op-amp circuit that can implement your controller. Label what the inputs and outputs of your circuit should be connected to.
- 5) Choose appropriate values for the resistors and capacitors in your circuit.

Mean 54
Std Dev 18

**MAE106 2002
Design Exam Solution**

The basic idea of this problem is to design a proportional/derivative (PD) position controller that is critically damped and moves to its final value in about 0.4 seconds in response to a step input.

- (20 pts) Control law: $\tau = -k_p(x - x_d) - k_d \dot{x}$
- (20 pts) Conveyor dynamics: $M\ddot{x} + B\dot{x} = \frac{\tau}{r}$

Controlled dynamics: $M\ddot{x} + B\dot{x} = \frac{-k_p}{r}(x - x_d) - \frac{k_d}{r}\dot{x}$

Rewrite in standard form: $M\ddot{x} + (B + \frac{k_d}{r})\dot{x} + \frac{k_p}{r}x = \frac{k_p}{r}x_d$

i.e. $M\ddot{x} + C\dot{x} + Kx = Kx_d$ where $C = B + \frac{k_d}{r}$ $K = \frac{k_p}{r}$

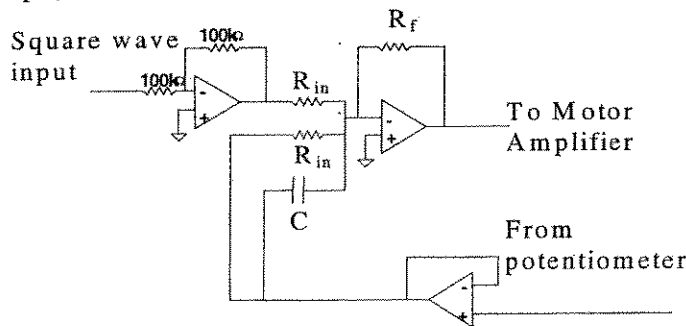
-- design system to be critically damped to push chip into board without overshoot $\zeta = 1$

-- design system so that it is within 2% of its final value in 0.4 sec $t_s = \frac{4}{\zeta\omega_n} = .4s$

thus, $\omega_n = 10 \text{ rad/s} = \sqrt{\frac{K}{M}} \Rightarrow K = 100 \text{ kg/s}^2 = k_p/r \Rightarrow \underline{k_p = 10 \text{ kg m/s}^2}$

now, $\zeta = \frac{C}{2\sqrt{KM}} = 1 \Rightarrow C = B + \frac{k_d}{r} = 2\sqrt{(100)(1)} \Rightarrow \underline{k_d = 1.9 \text{ kg m/s}}$

- (20 pts) I choose the damping ratio to be 1 (critically damped) so the system would not overshoot. I choose the settling time so that the system was within 2% of its final value at 0.4s.
- (20 pts)



- (20 pts) Choosing the values is made easy because 1 volt = 1 m on the inputs, and 1 volt = 1 Nm at the output to the motor amplifier (i.e. the calibration factors are equal to 1, and circuit gain values should just equal the controller gain values from the dynamics)
 $k_p = R_f/R_{in} \Rightarrow$ So if I choose $R_f = 100K\Omega$, then $\underline{R_{in} = 10 K\Omega}$
 $k_d = R_f C \Rightarrow \underline{C = 1.9/100K\Omega = .19 \mu F}$