Mechanical Systems Laboratory: Lecture 6
Integral Control; Introduction to Second Order Systems

1. Integral Control

In lab this week you are building an op-amp circuit for controlling velocity of a motor using proportional feedback. To place this lab in a context, we imagined in the last lecture that you are designing a velocity control system for "Robbie the Rescue Robot". We created a Proportional-type controller for Robbie, and found that the controlled system dynamics were as follows:

$$\underbrace{J\dot{\omega} = \tau}_{\text{dynamics}} \underbrace{= \alpha V}_{\text{current amplifier/motor}} \qquad \underbrace{V = -K(\omega - \omega_d)}_{\text{FB controller}} \quad \Longrightarrow \quad \frac{J}{\alpha}\dot{\omega} + K\omega = K\omega_d$$

where v= voltage input to current amplifier that powers Robbies motors, ω = actual angular velocity of wheels, sensed with a tachometer, ω_d = desired angular velocity, K = proportional feedback gain, alpha = proportionality constant relating v (i.e. current amplifier input) to torque output from motor.

Note that the steady-state error for this system is zero:

$$\dot{\omega} = 0 \text{ (steady state)} \rightarrow \omega = \omega_d$$

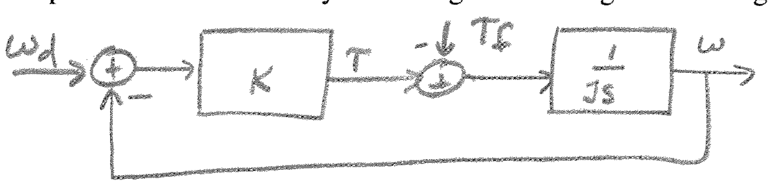
A more realistic model of Robbie's dynamics would include some friction in Robbie's wheels:

$$J\dot{\omega} = \tau - \tau_f \qquad \text{Assume } \tau_f = \text{constant (stiction)}$$

Let's assume that we control the torque to the motor directly, and express our control law in terms of torque. (Note, we actually control the current into the motor, but this is proportional to torque).

$$\tau = -K(\omega - \omega_d)$$

We can represent the combined system using a block diagram showing friction as disturbance.



Problem: Show that there is a steady-state error in velocity due to the friction.

$$\tau = -Ke \qquad e = \omega - \omega_d$$

$$J\dot{\omega} = -Ke - \tau_f \qquad \text{In steady state } \dot{\omega} = 0 \Rightarrow e = \frac{-\tau_f}{K}$$

KEY IDEA: We can get rid of this steady-state error by using a proportional plus integral (PI) controller:

$$\tau = -K_p e - K_I \int e\,dt$$

$$J\dot{\omega} = \tau - \tau_f = -K_p e - K_I \int e\,dt - \tau_f$$

$$\frac{d}{dt}: \quad J\ddot{\omega} = -K_p \dot{e} - K_I e \qquad \text{Not } \ddot{\omega} = \ddot{e} \text{ if } \omega_d = \text{constant}$$

$$\rightarrow J\ddot{e} + K_p\dot{e} + K_I e = 0$$

steady-state: $\dot{e} = \ddot{e} = 0$ $\Rightarrow e = 0$

$e = \omega - \omega_d$

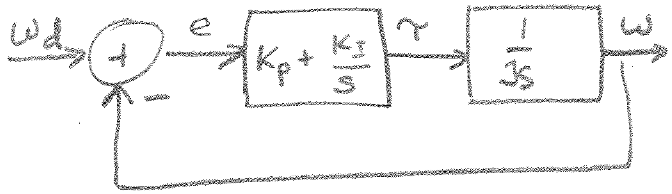
$\dot{e} = \dot{\omega}$

$\ddot{e} = \ddot{\omega}$

How does I control work? (try to explain it to your neighbor in words).

Integral control works in the following way:
If error e(t) does not equal zero, then $\int e(t)dt$ increases with time, and eventually the torque (which is proportional to this integral) becomes high enough to overcome friction.

The block diagram for a P-I compensator is:



What is the transfer function for this system?

$e = \omega_d - \omega$

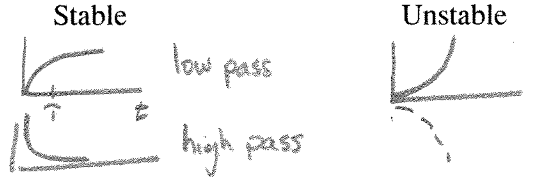
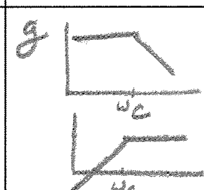
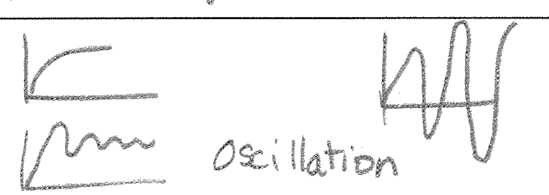
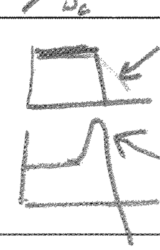
$\tau = \left(K_p + \frac{K_I}{s}\right) e$

$\omega = \frac{1}{Js}\tau = \frac{1}{Js}\left(K_p + \frac{K_I}{s}\right)(\omega_d - \omega)$

Solve for ω

$$\omega = \frac{K_p s + K_I}{Js^2 + K_p s + K_I}\,\omega_d$$

This is an example of second order system, which behaves differently than a first order system.

	Typical behaviors in time domain (step response)	Typical behaviors in frequency domain
First order system	Stable / Unstable  low pass, high pass	 low pass, high pass
Second order system	 Oscillation	 steeper cut off, resonance

Important Ideas: integral control can help remove steady state error. However, I-control adds dynamics to the system, which can lead to non-1st-order phenomena such as oscillation and resonance.