Name: __________________________
1. (20 points) Consider the diagram below. All pulleys are massless and frictionless. We would like to find a simpler equivalent system for the motion of the mass (i.e., for the same $x$). Find the equivalent spring, in terms of $K_1$ and $K_2$.

HINT: Use the equivalent displacement under similar force method, the energy method will messy.

Figure 1: Schematic of Problem 1
2 (25 points). Write the differential equation of motion in terms of \( x \), for the unforced motion of the system in the figure below, after it is disturbed by giving it an initial displacement and velocity. (You do not need to solve the equation.)

\[ U_c \]

\[ K \]

\[ M \]

\[ r_2 \]

\[ J_o \text{ is known} \]

\[ \text{No friction} \]

Figure 2: Schematic of Problem 2
3. (25 points) A uniform rod of length $2L$ and mass $m$, with a moment of inertia $I_{c.g.} = \frac{1}{12}m(2l)^2$ about its center of gravity, and $I_o = \frac{1}{3}m(2l)^2$ about point $O$, is hinged to the ground at point $O$ and supported by two springs and two dash-pots as shown below. The lower connections are at the mid-point of the rod and the vertical line is the static equilibrium line.

a. Write down the differential equation of motion for the rotational motion of the rod (due to given initial conditions), for the variable $\theta$ - the clockwise rotation from the vertical line. You can assume that the springs and dashpots remain horizontal (e.g., have roller connections to the walls).

b. Let $c_1 = c_2 = 0$ and $k_1 = k_2 = k$. What is the critical value of $k$, in terms of $L$ and $m$ – below which small disturbance to the rod would result in large motion (and maybe even fall). Explain very briefly.

HINT: Assume $\theta$ is small.

![Figure 3: Schematic of Problem 3](image-url)
4. (30 points) In the system below, \( y(t) \) is the absolute displacement of the massless plate on the left and it is known to be \( y(t) = Y \sin wt \).

a. Write down the differential equation of motion for the mass, in terms of \( z(t) \) its relative motion with respect to the plate.

b. Assuming \( y(t) \) to be the input and \( z(t) \) as the output, what would be the transfer function for this system? (i.e., what is \( H(s) = \frac{Z(s)}{Y(s)} \)?)

\[ y(t) = Y \sin wt \]

Figure 4: Schematic of Problem 4