Scattering of acoustic waves by a vortex

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The scattering of a plane acoustic wave by an axisymmetric vortex in two dimensions is investigated at low Mach number. Two asymptotic regions may be defined when the wavelength of the acoustic waves is much longer than the scale of the vortex. The solution is developed using matched asymptotic expansions, with the Mach number as the expansion parameter. The leading-order scattered wave field consists of two components, a dipolar wave arising from the interaction in the vortical region, and another component arising from the interaction in the wave region. Far from the vortex, the leading-order scattered field takes the form $i(\pi - \theta)e^{ikX} + \frac{1}{2} \cos \theta \cot \left( \frac{1}{2} \theta \right)(2\pi/kR)^{1/2} e^{i(kR-\pi/4)}$. This expression is not valid in a parabolic region centered on the positive $X$-axis, where a different asymptotic solution is developed; the two solutions match onto each other to give a leading-order scattering amplitude that is finite and single-valued everywhere, and vanishes along the $X$-axis. The next term in the expansion has a non-zero far-field response along the $X$-axis.