Experiments on the dynamics of droplet collisions in a vacuum

K. D. Willis, M. E. Orme

Abstract High-ordered experiments of binary droplet collisions in a vacuum environment are performed in order to study the collision dynamics devoid of aerodynamic effects that could otherwise obstruct the experimental observations by causing distortion or even disintegration of the coalesced mass. Pre-collision droplets are generated from capillary stream break-up at wavelengths much larger than those generated with the typical Rayleigh droplet formation in order to reduce the interactions among the collision products. Experimental results show that the range of droplet Weber number necessary to describe the boundaries between permanent coalescence and coalescence followed by separation is several orders of magnitude higher than has been reported in experiments conducted at standard atmospheric pressures with lower viscosity liquids (i.e. hydrocarbon fuels and water). Additionally, the time periods of both the oblate and prolate portions of the coalesced droplet oscillation have been measured and it is reported for the first time that the time period for the prolate portion of the oscillation grows exponentially with the Weber number. Finally, new pictorial results are presented for droplet collisions between non-spherical droplets.

1 Introduction

In this paper we seek to further the understanding of the dynamics of binary droplet collisions of a viscous fluid by performing the experiments in a vacuum environment. In this experiment, the background air pressure is typically 10^{-4} torr, corresponding to a number density of approximately 3 \times 10^{12}/cm^{3}, yielding an ambient density of approximately 1.4 \times 10^{-10} g/cm^{3}. The low-density environment will act to significantly reduce the aerodynamic effects that would otherwise obstruct the fundamental fluid dynamics of the collision phenomenon. Such experiments enable observations of the evolution of the collision dynamics as the deforming post-collision mass progresses downstream from the point of impact to distances up to 10^3 times the pre-collision droplet diameter, corresponding to approximately 25 ms travel time which is much longer than the travel times reported in other works involving droplet collisions in a standard ambient environment. Additionally, generation of the pre-collision droplets at extended wavelengths with non-conventional forcing techniques allows negligible interactions between successive post-collision products.

Previous works have characterized the possible outcomes of the droplet collision as a function of the kinetic energy of the collision, the size of the droplets, the impact parameter of the collision, and the material properties of both the droplet and ambient fluids. The droplet size ratio, which is defined as the radius of the smaller drop divided by the radius of the larger drop, is an additional parameter relevant in the case of collisions between unequally sized droplets. Additionally, previous workers in this field have identified that one of the most important dimensionless parameters for describing droplet collisions is the Weber number, We, which is defined for equal-sized drops of radius r_0 as:

\[
\text{We} = \frac{\rho (2r_0)U^2}{\sigma}
\]

where U is the relative velocity between the droplets, and \(\rho\) and \(\sigma\) are the density and surface tension respectively of the droplet fluid. Another important parameter for describing droplet collisions is the impact parameter \(b\), which is defined as \(\chi/(2r_0)\) where \(\chi\) is the projection of the separation distance between the centers of the colliding droplets normal to \(U\). Figure 1 provides a schematic of the collision geometry. The droplets’ radii are assumed to be different, denoted here as \(r_L\) and \(r_S\), where the subscripts L and S indicate the larger and smaller droplet respectively. The size ratio, \(\Delta\), of the drops is defined in this case as being equal to \(r_S/r_L\). If \(V_L\) and \(V_S\) are the velocities of the droplets, then \(U\), the relative velocity of the collision, can be determined from:

\[
U^2 = V_{L}^2 + V_{S}^2 - 2V_LV_S \cos(\theta_L + \theta_S)
\]

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where \((\theta_L + \theta_S)\) is the angle formed by the droplets’ trajectories. It is important to note that this is the relative velocity between the two droplets in the laboratory reference frame.

Previous studies have noted that when the droplet collision outcomes are viewed on the \(b\)-\(U\) plane, distinct regimes can be identified which can generally be categorized into four different types: bounce, permanent coalescence, coalescence with separation, and coalescence with shattering. The primary concern of the majority of recent studies in the field of droplet collisions is the identification of the critical values of \(W_e\) and \(b\) which determine the boundaries between the different types of collision outcomes.

The phenomenon of bouncing occurs when the kinetic energy of the collision is insufficient to expel the intervening layer of gas between the two pre-collision drops. The gas layer forms a physical barrier that prevents the surfaces of the drops from making contact. In this work we will not encounter the bouncing regime since the experiments are conducted in a vacuum, but rather will concentrate on the fluid mechanics of coalescence and subsequent separation.

Much of the early work conducted on droplet collisions was focused on understanding the mechanisms of raindrop formation, and therefore there is a vast amount of experimental data concerning water droplet collisions in a standard ambient environment. Abbot (1977) provides an excellent review of these studies. More recently, the study of droplet collisions has been found to be important for the understanding of secondary atomization processes in dense spray systems (e.g. Faeth 1977; O’Rourke and Bracco 1980). Additionally, a complete understanding of this phenomenon is relevant to the efficient distribution of agricultural sprays (Reichard 1997), and net-form manufacturing processes utilizing streams of molten metals (Orme 1993).

The study of Adam et al. (1968) is one of the more important works belonging to the earlier set which focused on water droplet collisions for the understanding of raindrop formation. They studied the collisional behavior resulting from the impact of equally sized water droplets (120 \(\mu\)m and 600 \(\mu\)m diameter drops) over systematic variations of both the relative impact velocity and the impact parameter. Their results are adapted in Fig. 2a and demonstrate that when the collision outcomes are mapped in this parameter space, distinctly different boundaries are observed for the two droplet radii cases tested. For 600 \(\mu\)m drops, collisions are stable in region (i), and unstable in

![Diagram of collision geometry](image-url)

**Fig. 1.** Schematic of collision geometry

![Diagram of coalescence boundary](image-url)

**Fig. 2.** a) Stable/unstable coalescence boundary in the \(b\)-\(U\) parameter space for 120 and 600 \(\mu\)m diameter drops from Adam et al. (1968). For 600 \(\mu\)m drops, collisions are stable in region (i), and unstable in regions (ii) and (iii). For 120 \(\mu\)m drops, regions (i) and (ii) represent the stable coalescence space and region (iii) is unstable b) Plot of the same data in the \(b\)-\(W_e\) parameter space. Here, stable coalescence for 120 \(\mu\)m drops occurs in region (i), and unstable coalescence occurs in regions (ii) and (iii). For 600 \(\mu\)m drops, regions (i) and (ii) represent the regime for stable coalescence, and region (iii) depicts the regime for unstable coalescence. Data points adapted from Adam et al.
regions (ii) and (iii). For 120 μm drops, regions (i) and (ii) represent the stable coalescence space and region (iii) corresponds to the unstable coalescence space. In region (iii), the high impact parameter separations are termed stretching separations and the low impact parameter separations are termed reflexive separations.

Another study in which attention was focused on defining the boundary between droplet separation and permanent union for water droplet collisions was conducted by Brazier-Smith et al. (1972). This work is among the first to employ a form of the Weber number (defined in their study as $U^2 r/p/\sigma$) in an attempt to make their results independent of drop size. However, it should be noted that different boundaries also result when the data due to Adam et al. (1968) shown in Fig. 2a is mapped in the b-We plane as shown in Fig. 2b.

A large portion of the early work on water droplet collisions including that due to Brazier-Smith (1972), Beard et al. (1979), and Low and List (1982) examined the probability that a collision event will result in a permanent coalescence, and therefore they defined empirical relations for the coalescence efficiency. In many cases, the coalescence efficiency was determined based on the number of post-collision particles.

More recent investigations have focused on the collisional behavior of hydrocarbon fuels for application to combustion systems (Ashgriz and Givi 1987, 1989; Brenn and Frohn 1989a, b; Jiang et al. 1990; Qian and Law 1997). The research by Jiang et al. (1990) yielded results that have been recast in Fig. 3, illustrating the regimes of droplet collision outcomes shown on the b-We plane. Regimes I through V represent the regions of coalescence, bounce, coalescence, reflexive separation, and stretching separation, respectively. They observed fundamental differences between the impact behavior of water and hydrocarbon fuel droplets, which can be summarized by noting the existence of a bouncing regime for hydrocarbon droplets at ambient pressures. Subsequent work by Qian and Law (1997) was successful in qualitatively unifying the previous observations for water and hydrocarbon fuel droplet collisions by showing that the five collision outcome regimes previously observed for hydrocarbon fuel droplets could also be observed for water droplets by varying the density of the ambient gas through its pressure and molecular weight.

A more complete discussion of the research to date on the binary collision process for water and hydrocarbon droplets can be found in the review article by Orme (1997).

In the above works [with the exception of Qian and Law (1997), who also studied droplet collisions at reduced pressures], experiments were carried out in a standard ambient environment or higher background pressures, thereby limiting the information that is free of aerodynamic effects. In an effort to provide a “cleaner” description of the underlying physical phenomenon, the authors of this research have taken the unique approach of studying the collisions of low vapor pressure oil droplets in a vacuum environment. The main advantage of this approach is that the droplet dynamics are completely decoupled from aerodynamic effects.

2 Experimental

The main component of the experimental setup is a 43 cm × 43 cm × 91 cm vacuum chamber which is mounted with the major axis aligned vertically on a vibration isolation table (see apparatus schematic, Fig. 4). Chamber pressures of $10^{-4}$ torr can typically be attained through the use of a diffusion and mechanical pump system. The chamber is constructed out of a welded aluminum skeleton with Plexiglas windows, which allow optical access to over 50% of the vertical chamber area.

Two droplet generators, mounted on 45 cm flight tubes, are connected to the top of the chamber via an inter-changeable beveled flange. The droplet generators employ a piezoelectric-orifice design, described in more detail by Orme and Muntz (1990), which allows for the generation of spatially and temporally stable droplet streams. Controlled droplet collisions are produced within the chamber at the intersection point of the streams. Adjustment of the collision angle is achieved through the use of different beveled flanges. The socket/hemisphere design of the droplet generators along with two micro-translators mounted in orthogonal directions to each other on each flight tube allows precise adjustments to be made to the positions of the streams. Additional considerations have been made in the design of the collision chamber (i.e. the additional flight tubes), which allow for the generation of droplet streams at wavenumbers extended over those which can typically be achieved with conventional Rayleigh forcing, using a technique first described by Orme and Muntz (1987) and described in Sect. 2.1.

Because of the vacuum conditions desired, options for the droplet generator working fluid are restricted. For this study Dow Corning 200 fluid was selected based on its vapor pressure of $10^{-3}$ torr at 25 °C. Other fluid properties as reported by the manufacturer are a density of 0.949 g/cm³, surface tension of 20.6 dynes/cm, and a viscosity of 30 cSt. Orme and Muntz (1990) have previously found that the capillary stream break-up length is in good agreement with prediction, leading to the assumption that the surface tension does not change significantly in a vacuum environment. A fluid pump located below the fluid reservoir at the bottom of the chamber is used to circulate the fluid.

![Fig. 3. Schematic of the five droplet collision outcome regimes on the b-We plane adapted from Jiang et al. (1990). Regime I – coalescing, regime II – bouncing, regime III – coalescing, regime IV – reflexively separating, regime V – stretching separation](image-url)
back into the chamber for out-gassing or up to the accumulator/droplet generators for running experiments. It was found that significant attention must be paid to provide a fluid free of embedded gasses, since failure to do so will cause droplet stream generation with high speed and angular dispersions due to the fact that the fluid with embedded gases will cavitate when exposed to the vacuum environment.

Control of the droplet stream speed and therefore the relative impact velocity is achieved by regulating the pressure driving the flow through the orifice. This was accomplished though the use of a fluid accumulator located near the top of the apparatus. The fluid accumulator consists of a pressure vessel which has two chambers separated by a flexible rubber diaphragm. One chamber is used as a droplet fluid reservoir, while the other is connected to a regulated nitrogen supply. This allows the driving pressure to be isolated from the droplet fluid, and additionally eliminates surges and mechanical vibrations from the re-circulation pump.

2.1 Droplet generation

Droplets in this study are generated by the well-known mechanism of capillary stream break-up. In order to extend the separation between droplets in an effort to isolate the successive collision products, the droplet stream was manipulated with the method described by Orme and Muntz (1987) and will be described in brevity later. To form droplets with capillary stream break-up, a disturbance is imposed on the stream that initiates the axisymmetric instability on the stream’s radius. If the disturbance is such that the nondimensional wavenumber, $k_0^d$, is less than unity, where $k_0^d = 2 \pi r_0 / \lambda$, $\lambda$ being the wavelength of the imposed disturbance, the instability will grow until droplets ultimately pinch off of the stream. Lord Rayleigh (1879) showed in his linear stability analysis that the disturbance grows on the stream as $e^{\beta t}$ where $\beta$ is the growth rate that depends on the initial stream radius, forcing ($k_0^d$), and fluid properties of density and surface tension. Weber and Agnew (1931) subsequently developed an expression for $\beta$ which includes the effects of the viscosity of the fluid. It has been shown by Orme (1991) that the stream has the lowest speed dispersion at the nondimensional wavenumber of maximum growth rate.

In this work, the Weber number is varied by changing the driving pressure of the droplets, thereby changing the relative impact speed. While changing the pressure, care has been taken to hold $k_0^d$ constant throughout each experimental realization. Using conservation of mass, the droplet radius, $r_d$, can be estimated as:

$$ r_d = r_0 \left( \frac{3}{2} \frac{\pi}{k_0^d} \right)^{1/3}. $$

Giving a value of the radius, $r_d = 170 \mu m$ for $k_0^d = 0.4$, and $r_0 = 75 \mu m$.

To extend the separation between successive droplets, an amplitude modulated disturbance is imposed on the capillary stream, where the frequency ratio, $N$, is the ratio of the fast frequency (carrier frequency) to the slow frequency (modulation frequency). The physics of the droplet formation process subject to amplitude modulated forcing
has been described in detail in previous work by Orme (1990) and will be described briefly here.

The droplets formed from an amplitude modulated disturbance will be generated at initial separations commensurate with the wavelength of the carrier disturbance, $\lambda_c$ similar to the case of droplet generation with non-amplitude modulated disturbances. Unlike the latter, however, the droplets generated with amplitude modulated disturbances will have predictable relative velocities resulting from the imposed modulation. The relative velocities will cause the carrier droplets to systematically coalesce in flight to form a final stream of droplets which are separated a distance commensurate with the modulation frequency. It has been shown in the previously referenced works that the speed dispersion of the final droplet stream varies as $\sigma_v/N$, the drop-to-drop separation between droplets varies as $N\lambda_c$ and the droplet diameter $r_m$ varies as $r_m = r_d N^{1/3}$, where $\sigma_v$ and $\lambda_c$ are the speed dispersion and wavelength of the droplet stream generated with a conventional unmodulated sinusoidal disturbance, respectively. It should be noted that the previously mentioned relationships are applicable to fully merged droplet streams where $N$ is restricted to be of an integer value.

To insure a highly uniform droplet stream, the carrier frequency is chosen such that $k_{01}^* = N \pi \lambda_c$ is in the region of maximum amplitude growth rate. Hence, for $r_d = 170 \mu m$ as given above, and $N = 4$, the pre-collision droplet radius $r_m = 270 \mu m$, generated from an orifice radius of 75 $\mu m$. It should be noted that use of the amplitude modulated disturbance for the generation of pre-collision droplets is not feasible in a standard ambient environment or one at higher pressures. This is because a finite flight time is required for the original droplets to coalesce into modulation drops, where the amount of time depends on the frequency ratio $N$ (the higher the $N$ the longer the time), the amplitudes of the carrier and the modulation disturbances, and the phase difference between the two disturbances. If the droplets were caused to travel long distances in a higher pressure environment, they would experience significant speed and angular dispersions due to aerodynamic effects and would not be suitable for controlled collision studies.

3 Experimental results

3.1 General phenomenon

Figure 5 shows the typical droplet deformation process that results from a head-on impact. We have identified two separate regimes, shown as $R_t$ and $R_{II}$, which occur after a binary collision and are characterized by different physical geometry and exhibit distinct deformation behavior. It is natural to think of $R_t$ as the oblate regime whereas $R_{II}$ can be considered the prolate regime. In this example $We = 1560$ and the time elapsed between time $t = 0$ and $t = t_{2x}$ is 9.0 ms. Droplet fission, of course, can occur in either $R_t$ (shattering collisions) or $R_{II}$ (reflexive separation). Shattering collisions have been found to occur if the initial kinetic energy of the collision is sufficiently high.

Fundamental investigations by Harlow and Shannon (1967) have studied this phenomenon in their work on splashing. However, a detailed parametric study of the critical conditions necessary for shattering to occur relevant to binary droplet collisions with size ratio of $O(1)$ have not been conducted for any fluid to the knowledge of these authors.

Insight may be drawn from the widely investigated case of impinging jets to which droplet collisions can be loosely viewed as a time-dependent analogue. For impinging jets, the sheet disintegration mechanism has been attributed to
Taylor cardiodal waves or the growth of Kelvin–Helmholtz instability waves depending on the magnitude of the Weber number (Huang 1970; Ibrahim and Przekwas 1991). It is thought that these mechanisms are related to pressure fluctuations in the stream near the point of impingement and shear on the surface of the forming sheet due to the velocity difference between the fluid and the surrounding medium. In this work, the droplets travel up to 3000 droplet diameters after generation and prior to collision, leading to the assumption that any internal pressure fluctuations within the drop are negligibly small due to damping. Because of the vacuum conditions employed in this study, Kelvin–Helmholtz instabilities are not expected to develop. Over the range of Weber number tested in this study (761 < We < 3200), shattering collisions have not been observed, therefore no further conclusions can be drawn concerning this phenomenon.

Our experimental data consists of a series of digital images of the droplet deformation for a head-on impact over a range of Weber numbers. The pre-collision droplet diameter is not varied and has a measured value of 540 µm ± 35 µm (the uncertainty of ±35 µm corresponds to the pixel width of the digital images) in excellent agreement with the predicted value discussed earlier. As shown in Fig. 5, the deformation subsequent to the impact results in the formation of a circular disk oriented in a plane orthogonal to the original relative velocity vectors of the pre-collision drops. The oblique view shown here is from about 15° out of this plane.

Immediately subsequent to the droplet impact at $t = 0$, a high-pressure region is formed around the stagnation point between the two drops. The implications are two-fold: the incoming fluid velocity is reduced and the fluid already in the impingement zone is accelerated out radially. This is an unsteady flow problem and the characteristic time for the duration of this pressure field is on the order of $4r_0/U$ which is just the time taken by a drop to move one diameter ($2r_0$) at half the relative impact velocity ($U/2$). Once the pressure field has subsided, the interaction between viscous, surface tension, and inertial forces determines the deformation dynamics until nearly the end of the regime I oscillation where again a transient pressure field arises due to the impinging flow situation.

As can be observed in Fig. 5, the radially expanding circular disk formed after the collision develops a rim at some point between $t = 0$ and $t = t_{n/2}$. The region of high curvature at the tip of the expanding disk has associated with it a region of high pressure due to surface tension, which results in the acceleration of the fluid within that region back towards the center of the disk. Phenomenologically, this behavior is very similar to recent descriptions of extended fluid ligament dynamics by Stone and Leal (1989) with the only major difference being the axis of symmetry.

At $t = t_{n/2}$ the deformed coalesced mass, which now closely resembles a torus connected throughout the interior by a thin fluid sheet, reaches its maximum extension and begins contracting back towards the center under the action of surface tension. In the thin fluid sheet region, the almost parallel free surfaces imply that capillary forces can be neglected, leaving only inertial and viscous forces to determine the fluid behavior in this region. This implies that the fluid in this region is unaware of the dynamics of the rim. This assertion appears to be consistent with the previous experimental findings of Ranz (1959) in his research on the dynamics of thin films. The postulate that the fluid in the sheet is convected into the rim in a continuous fashion throughout the regime I deformation process is supported by the observations shown in Figs. 5 and 6, in that the percentage of the droplet mass contained in the rim appears to be increasing throughout the regime I deformation.

![Fig. 6. Image showing sequence of coalesced droplet deformation. Each coalesced droplet is separated by a time increment equal to approximately $1/f_d$, where $f_d$ is the droplet generation frequency](image-url)
The impinging flow resulting from the collapse of the rim just prior to \( t = t_n \) generates a high pressure region around the stagnation point at the center of the disk. Again this has two effects: slowing of the fluid still in the rim and rapid acceleration of the fluid already in the impingement zone. The result is the formation of two jets which act opposite in direction to the original velocity vectors of the drops. These jets result in the formation of a diverging drop pair connected by a fluid ligament, which attains a maximum deformation at \( t = t_{3\pi/2} \) in the case of fusion. The fission event will be discussed in Sect. 3.3.

The droplet deformation is observed to exhibit similar general behavior over the range of Weber numbers between 760 and about 2840. For conditions employed in this study, collisions within this range always result in permanent coalescence. As noted previously and shown in Fig. 2, the critical Weber number which defines the transition between stable coalescence and coalescence followed by separation is on the order of 100 for water drop collisions in a standard ambient environment. In addition, the magnitude of the maximum deformations at \( t = t_{\pi/2} \) and \( t = t_{3\pi/2} \) are observed to increase monotonically with the Weber number. Permanent droplet fission is observed to occur above a critical Weber number of approximately 2840, which is an order of magnitude higher than that for water drop collisions in a standard atmosphere. It is thought that this increase in critical Weber number is due to the fact that there is negligible air pressure to aerodynamically disrupt the thin films or ligaments generated in the collision. Figure 7 depicts the phenomenological trends discussed here. It is noted that a complete time history of the droplet deformation process is not possible given the structural constraints of the vacuum chamber; however, this should not distract from the results and conclusions presented here.

### 3.2 Time periods for regime I and II

In addition to the digital images, measurements were taken of the half-oscillation periods \( t_{RI} \) and \( t_{RII} \), which are defined with the aid of Fig. 5 as:

\[
\begin{align*}
t_{RI} &= t_n \\
t_{RII} &= t_{2n} - t_n
\end{align*}
\]

(2a)

(2b)

In Fig. 8, the measured time periods for regime I and II are shown plotted against the Weber number. Although \( t_{RI} \) appears to be a constant with \( We \) as can be seen from Fig. 9a, it is actually a weak function of the Weber number and a least squares minimization linear curve fit of the data yields the following expression:

\[
t_{RI} = 0.00175 + 1.25 \times 10^{-7} We
\]

(3)

When plotted on a log-linear scale, \( t_{RII} \) is seen to vary linearly as shown in Fig. 9b, therefore the regime II time period has been empirically determined to be of the form:

\[
t_{RII} = C e^{a We}
\]

(4)

where \( a \) represents a linear growth rate, assumed to vary with fluid properties, and is the subject of ongoing work. It should be noted that this is the first time that \( t_{RII} \) has been reported to vary exponentially with \( We \), and the ability to acquire the measurements described here relies on the

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**Fig. 7. Summary of droplet deformation behavior**
controlled propagation of the collision products in a vacuum environment.

Differences between the time a droplet spends in the prolate and oblate configuration have been noted previously in both numerical studies by Foote (1975) and experimental work on acoustically levitated droplets by Trinh and Wang (1982). However, to the authors’ knowledge, this is the first time detailed measurements have been conducted which demonstrate the distinct functional relationships of the oblate and prolate portions of the oscillation to the Weber number associated with the droplet deformation as demonstrated in Fig. 8 and Eqs. (3) and (4).

3.3 Droplet fission
The deformation dynamics of the merged droplet mass through the first half of the regime II oscillation results in the formation of an extended fluid ligament connecting two end-drops. The only means whereby momentum can be transferred between the two connected masses is through the ligament where the instantaneous force acting on each half of the droplet mass can be written as

\[ F = 2\pi r \sigma \]

(5)

where \( r \) is the minimum radius of the ligament connecting the end-drops. Although this assumes that the minimum dimension occurs at the center of the ligament, which is not necessarily the case, it appears to be a reasonable assumption since the necks formed on the ligament appear to break nearly simultaneously. The criteria that must be evaluated in order to determine whether a collision outcome will be permanent fusion or fission is the momentum condition of the end-drops at the instant the fluid ligament connecting them breaks up.

Under the controlled conditions of a vacuum chamber, i.e. in the absence of external forces, there are only two possible outcomes. If the fluid ligament breaks before the outward momentum of the drop pair is arrested; then the result is the formation of at least two droplets. Alternatively, if the momentum of the droplet pair is arrested prior to the break-up of the fluid ligament, then the eventual result, given enough time and in the absence of external forces, is permanent fusion although there may be a temporary fission.

Figure 10 is an image of the break-up of the extended fluid ligament due to capillary instabilities at \( t = t_{3x/2} \) for a \( \text{We} = 2840 \) collision. What is observed for these conditions is that the momentum transfer is such that the
end-drops separate from the fluid ligament with nearly no momentum. The result is that their relative lateral position appears to beunchanging with respect to time. At a slightly higher Weber number (=2880) the droplets are observed to be diverging from the center of mass after the break in the fluid ligament while at a slightly lower Weber number (=2800) they are converging.

3.4 Radial rim development measurements
An experimental analysis of the spreading rim region was undertaken in order to gain insight into the dynamics of the deformation process. The images from which the data are extracted typically contain 6–7 droplet deformation events (see example shown in Fig. 6) separated by an increment of time approximately equal to the inverse of the droplet generation frequency, $f_d$. The previous findings of Orme and Muntz (1987) showed that a speed dispersion of less than $10^{-5}$ times the average droplet speed can be expected under the vacuum conditions in which these results are attained. Their results imply that the time separating each successive deformed droplet is equal to $1/f_d$ with a standard deviation equal to $\sim2\mu$s.

Figure 11 presents experimental data of the radial position of the rim as a function of time for five different Weber numbers. The symbols $\square$, $\triangle$, +, and ■ correspond to $We = 761$, 1019, 1560, 1840, and 2143, respectively, and the lines through the data are curve fits which are described below. Differentiation of the curve depicting the radial expansion of the rim with respect to time yields the velocity shown in Fig. 11b, and differentiation twice yields the acceleration experienced by the rim shown in Fig. 11c. It is seen that increasing the Weber number has the effect of increasing the diameter of the rim over a fixed time duration. Also, higher Weber numbers correspond to higher velocities and accelerations in the rim.

Our goal in studying the rim deformation dynamics is to characterize the rim behavior in terms of the Weber number of the collision. It should be noted however, that a precise analysis of this problem would require a computationally intensive simulation of the Navier–Stokes equations. What we seek to illustrate with the following exercise is an expression for the rim radius as a function of Weber number only. The general equation used to fit the experimentally obtained radial position of the rim is a third-order polynomial and is given by:

$$R(\hat{t}) = A_1(\hat{t}) + A_2(\hat{t})^2 + A_3(\hat{t})^3$$  \hspace{1cm} (6)

where $R$ is the radial position of the outermost edge of the rim, $\hat{t}$ is the transformed time, and $A_1$, $A_2$, $A_3$ are coefficients to be determined. The transformed time is related to the experimental time, $t$, by the equation:

$$\hat{t} = t + t_0$$

where $t_0$ is the magnitude of the shift along the time axis which produces the best fit of the experimental data to the third-order polynomial.

We assume that the initial velocity of the developing rim is equal to half the relative velocity of the drops at impact, which is equivalent to writing:

$$\frac{d(R)}{dt}{\bigg|}_{t=-t_0} = \frac{U}{2} = A_1.$$ \hspace{1cm} (7)

Now, if we also require that the radial position at time $t = t_\pi$ ($\hat{t} = t_\pi + t_0$) to be equal to the radius of the merged drop, $R_{md}$, we can solve for $A_2$ as follows:

$$A_2 = \frac{R_{md} - \frac{U}{2}(t_\pi + t_0)}{(t_\pi + t_0)^2} - A_3(t_\pi + t_0).$$ \hspace{1cm} (8)

Remembering that $U$ is just a function of Weber number and using the empirically determined linear relationship for $t_\pi$ with Weber number given by Eqs. (2) and (3), Eq. (6) can be reduced to just two unknowns, $A_3$ and $t_0$. A least-squares minimization curve fit through the data was
used to determine the parameters $A_3$ and $t_0$ from the radial data at each Weber number. Figure 12 is a plot of the variation of $A_3$ versus Weber number. The scatter on both sides of the solid line is postulated to be representative of the error in the measurements. Hence, with the utilization of the value of $A_3$ given by the solid line in the third-order polynomial curve fit described above, predictions of the radial expansion of the rim can be obtained for any Weber number. As can be seen, the parameter $t_0$ is nearly constant for all the curves.

### 3.5 Collisions between non-spherical droplets with non-zero impact parameters

In this section we present collision results from the use of non-spherical pre-collision droplets. While these results are fundamentally interesting in their own right, it is recognized that practical usefulness of such collisions is uncertain at this time. Non-spherical pre-collision droplets are obtained by imposing an amplitude modulated disturbance on the capillary stream thereby initiating droplet formation as discussed in Sect. 2.1. Recall that the pre-collision droplets coalesce in flight to form the final stream of droplets. At some point in their flight the droplets will almost be merged. That is, they may have made contact but have not relaxed into a spherical form, but rather still maintain a pseudo dumb-bell shape. It is at this point in the development of the pre-collision droplet stream that they collide with another stream which may be selected to be spherical or non-spherical.

**Fig. 12.** Plot of the variation of the curve fitting coefficient $A_3$ versus Weber number

Figure 13 is a photograph of a droplet collision generated from two droplet streams which have not fully merged into modulation droplets. The pre-collision droplets are formed from three droplets with radii 155 µm each. Two of the three droplets have completely merged into one larger sphere and the third droplet is in the process of merging with the larger sphere at the time of collision with the similar stream. Hence the pre-collision droplets are characterized by uneven “dumb-bell” shapes. The collision product in this realization has been termed the “amoebae,” and is characterized by a fine fluid ligament on each end of the collision product.

**Fig. 13.** Example of collisions generated with non-spherical pre-collision droplets. Note the existence of thin fluid ligaments on each side of the collision product

**Fig. 14.** Example of collisions generated with non-spherical pre-collision droplets. Collision products resemble three-dimensional saddle

**Fig. 15.** Example of collisions generated with non-spherical pre-collision droplets. Collision products resemble hummingbirds when viewed from right to left
Figure 14 illustrates the collision product which has been termed the “saddle.” As can be seen the stream on the left is fully merged into modulation droplets while the droplets comprising the stream on the right have not fully relaxed into spherical modulation droplets. The resulting collision product is a three-dimensional thin film which appears to wrap around like a saddle.

Figure 15 illustrates the “humming-bird” collision, which is termed so due to the likeness of the collision product to humming-birds when the streams are viewed to travel from right to left. In this realization, neither of the droplet streams have relaxed into streams of modulation drops. Also, as can be seen in the photograph, the left stream leads the right stream which causes the shearing action of the streams to stretch the fluid in the front and rear of the collision product.

Figures 13–15 are just a few examples of the complicated nature of droplet collisions with non-spherical pre-collision droplets. Remarkably thin films often characterize the collision outcomes and ligaments that would become severely distorted or even disintegrate in a standard environment. The collision outcome is extremely sensitive to impact parameter and pre-collision droplet shape. It should be noted however, that once the desired collision outcome is obtained through variation of the input parameters, the experiment is extremely well-behaved in the sense that identical collision products are generated in a repeated fashion. It should be mentioned here that these experiments are not possible to conduct in a standard ambient environment because a finite flight distance is required to merge the pre-collision droplets, and then subsequent to the collision, the thin films and ligaments generated would become severely distorted in a non-vacuum pressure environment.

4 Summary

New experimental results have been reported for droplet collisions in a vacuum environment. The pre-collision droplets are generated with the use of unconventional forcing techniques that extend their separations, thereby limiting the interactions between successive droplet collisions. The evolution of the collision product is observed after it has traveled up to $10^5$ droplet diameters, which is only possible due to the rarefied background environment.

It has been shown that the collisions always result in permanent coalescence for $W$ up to 2840 which is over an order of magnitude higher than results reported by others for water droplet collisions in a standard environment. Furthermore, we have found that the duration of time that the post-collision mass spends in the second-half of the oscillation period varies exponentially with $W$, which is the first time such a result has been reported.

Additionally, new data of the dynamics of the spreading rim have been reported and have been empirically shown to depend only on Weber number, $W$ and time, $t$, where our measurements have been fit with a third-order polynomial in time and constant coefficients which depend only on $W$.

Finally, new results depicting the unusual characteristics of collisions between non-spherical droplets have been shown pictorially where the thin films and ligaments generated are highly sensitive to the input parameters and rely on the non-perturbing effects of the vacuum environment for their development.

Each of the new results presented here necessarily rely on the generation, propagation, and collision of the droplets in a rarefied background environment in order to capture the un-perturbed development of the dynamics of the droplet collision.

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