A. The duration of this exam is two hours.

B. Write your name on the blue book, your 6 pages of notes and the exam you are given. ALL THREE WILL BE COLLECTED AS THE END OF THE EXAMINATION

C. Once you are done, leave the exam on the seat and leave. The TA will collect the exams. All students must leave at the end of the two-hour period.

D. Do not waste time by rewriting the problem statement, or redrawing the figures, in the blue book. If you write comments on the figures, make sure you refer to it in your solution (to draw the instructor’s attention).

E. Grading scheme:
   - Problem 1: 20 points
   - Problem 2: 20 points
   - Problem 3: 30 points
   - Problem 4: 30 points

Name: ______________________________
1. (20 points total) Consider the figure below. The mass $m_2$ and stiffness $k_2$ are to be designed so that the $m_2$ and $k_2$ act as a **perfect vibration absorber** for the motion induced by the forcing function (which is applied at half way point between the hinge and right corner). All units are SI (i.e, Newtons, meters, kilograms, etc)

**a. WITHOUT** writing the equations of motion, can you come up with a good value for $\frac{k_2}{m_2}$? Explain briefly. (10 points)

**b.** What is the minimum mass for $m_2$ if you are told that the motion of $m_2$ should be less than 5 centimeters? (10 points)

![Figure 1: The Schematic of Problem 1](image-url)
2 (20 points total). a. Write the differential equation of motion, for the figure below, in terms of $x_1$, $x_2$, and $\theta$ as shown on the Figure (all absolute motions from the static equilibrium line). Write the equations in the matrix form.

Figure 2: The Schematic for Problem 2
3. (30 points total) In this Figure, the total length of the rod is $2L$ and its mass is $M$. Using the absolute vertical motion of the center of the rod, the rotation of the rod (clockwise) and the absolute motion of the masses (as shown in the Figure) as the degrees of freedom,

(i - 25 pts): write the equations of motion in the matrix form; i.e.,

$$M \ddot{x} + K x = 0$$

(ii - 5 pts): Let $K_1 = K_2 = K_3 = K_4 = K$ and $M = m_1 = m_2 = m$. Suppose you have been able to observe a mode shape of the form

$$X = (1 \ 0 \ 1 \ 1)^T$$

what is the natural frequency associated with this mode shape? Why?

HINT: Assume $\theta$ is small.

Figure 3: The Schematic for Problem 3
4. (30 points total) Consider the following chemical engineering problem. A molecule is modeled as 6 masses (atoms) connected by springs (chemical bonds). The molecule has a simple uniform structure, so all masses are $m$, and all stiffnesses are $k$. (see Figure A on the next page)

The problem is to break this molecule into two identical molecules, each with three atoms, i.e., the two molecules of the form shown in Figure B. The game plan is to use lasers to inject energy and excite the molecules such that they break up into halves. What we really need is to choose a laser frequency that results in excitation that stretches the middle spring and leaves the rest of the springs alone (if other springs are stretched too far, the molecule might break up into several smaller parts).

a. MODELING (20 points): Due to the one free bond at each end, the laser excitation can be modeled as a harmonic forcing function (at the frequency of the laser) applied to the end atoms. Write down the equations of motion for the six mass model. You need to identify $M$, $K$, $F$ in

$$M\ddot{x}(t) + Kx(t) = F(t)$$

For $x(t) = [x_1(t) \ x_2(t) \cdots x_6(t)]^T$ where $x_i(t)$ is absolute displacement of the $i^{th}$ mass.

The bone-headed chemists suggest the following course of action: An exhaustive trial and error approach in which lasers at all frequencies are tested, one after another, to see which one gives a better result. A former researcher had done a preliminary eigenvalue analysis of the equations of motion and obtained the following sets of eigenvalues and (normalized) eigenvectors

$$w_1 = 0.0 \text{ rad} \quad X^1 = [+0.4082 \ +0.4082 \ +0.4082 \ +0.4082 \ +0.4082 \ +0.4082]^T$$
$$w_2 = 0.517 \text{ rad} \quad X^2 = [-0.5577 \ -0.4082 \ -0.1494 \ +0.1494 \ +0.4082 \ +0.5577]^T$$
$$w_3 = 1.000 \text{ rad} \quad X^3 = [-0.5000 \ +0.000 \ +0.5000 \ +0.5000 \ +0.0000 \ -0.5000]^T$$
$$w_4 = 1.414 \text{ rad} \quad X^4 = [+0.4082 \ -0.4082 \ -0.4082 \ +0.4082 \ +0.4082 \ -0.4082]^T$$
$$w_5 = 1.731 \text{ rad} \quad X^5 = [-0.2887 \ +0.5577 \ -0.2887 \ -0.2887 \ +0.5577 \ -0.2887]^T$$
$$w_6 = 1.931 \text{ rad} \quad X^6 = [-0.1494 \ +0.4082 \ -0.5577 \ +0.5577 \ -0.4082 \ +0.1494]^T$$

b. (10 points) Do you agree with the chemists’ approach? Considering the mode shapes, can you come up with a few good candidate frequencies that excites the bond we are interested in breaking? Since we like to reduce excitation in the rest of the bonds, can you rule out any of your candidates? Explain the logic behind your answers as completely as you can. Do not rewrite Chapter 6. Simply justify your idea using what you have learned. Be clear and use general equations (such as the decoupled form of equations of motion) to substantiate your idea.
Fig. A: Original Molecule

Fig. B: Molecule Broken into Halves