A Couple of Hints:

1. There are five problems, and a total of 120 minutes.

2. The derivative of a rotating unit vector can be found via the following

\[ \frac{d}{dt} \hat{e} = \mathbf{W}_e \times \hat{e} \]

where \( \mathbf{W}_e \) is the angular velocity with which the unit vector \( \hat{e} \) rotates.

3. The total kinetic energy of a rigid body in translation and rotation is

\[ T = \frac{1}{2} m v_G^2 + \frac{1}{2} I \omega^2 \]

4. For a disk (gear or roller) the second moment of inertia around the center of gravity is \( \frac{1}{2} m r^2 \) and for a uniform rod it (i.e., \( I \) about the center of gravity) is \( \frac{1}{12} ML^2 \)
1 (15pt). In the figure below, the top rack is moving to the left at a velocity of 4 feet per second and the bottom rack is moving to the right at a velocity of 8 feet per second.

a. What is the angular velocity of the gear in the middle.

b. What is the velocity of the center of gear (i.e., point A)
2 (20 pt). In the diagram below, the circular disk $B$ of radius $r$ rolls without slipping in a circle of radius $b$ on a fixed surface $C$.

The central axis rotates about the vertical axis at a constant angular velocity of $\omega_0$.

a. Write expression for the total angular velocity of the disk $B$.

b. Is the angular velocity in part ‘a’ constant? If not, what is its derivative.

Hint: Introduce a rotating coordinate frame, if you need one. Identify it carefully!
3 (20 pt). In the figure below, the gears and the arm OA are on a flat horizontal plane (i.e., ignore gravity). The big gear (●) in the middle is fixed (it does not move, it does not rotate, it does not dance, it just sits there!). At the beginning, everything is at rest as shown.

Arm OA can be considered a uniform rod of length 3r and mass $M_{OA}$, while the gear $AB$ has radius $r$ and mass $m$. A torque $M$ (clockwise) is applied to rotate the arm OA (and with it the small gear $B$) to the top point (point $A'$). Calculate the angular velocities of the arm $AB$ and gear $B$ as point $A$ goes through the point $A'$.

What would you change if this was in the vertical plane (explain very briefly - only 5 points worth)
4 (20 pt). A box with mass $M$ is supported by 4 roller bearings each with mass $m$ (the figure below shows the two on one side). The rollers are not touching the plate and only their centers are welded to connecting rods (which themselves are welded to the plate).

A force $P$ is applied and it is observed that the rollers do not slip on the surface.

a. Write the total kinetic energy in terms of $M$, $m$ and $v$ - the velocity of the plate.

b. What would be the plate velocity after the force $P$ has moved it $d$ meters. Write the answers in terms of $M$, $m$, $K$, $P$ and $d$.  

![Diagram of a box with a force applied and rollers supported on a plate]
5 (25 pt). A uniform slender rod wit mass $m$ and length $l$ (and $I = \frac{1}{12} ml^2$) is confined so that its ends move in the horizontal and vertical slots shown. The spring has a stiffness $k$ and is unstretched when $\theta = 0$. Neglect the mass of the blocks and ALL FRICTIONS. The system is released from rest from the position of $\theta = 30$. Find

a. The total kinetic energy of the rod, in terms of the angular velocity of the rod ($\omega$), $m$, and $l$.

b. For any $\theta$, find the total mechanical energy of the system in terms of $k$, $l$, $m$, $g$ and $\theta$

c. Find the angular velocity of the rod when $\theta = 0$. What happens if $k$ is too small? What can it mean? Explain very briefly.